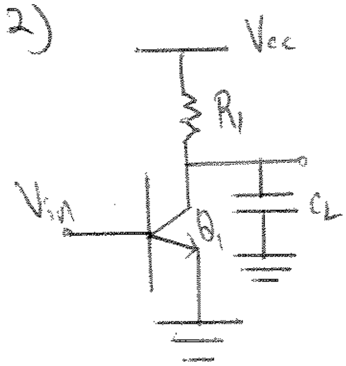
11.1

$$\begin{aligned}
\frac{V_{out}}{V_{in}}(j\omega) &= -g_m\left(R_D \parallel \frac{1}{j\omega C_L}\right) \\
&= -\frac{g_m R_D}{1 + j\omega C_L R_D} \\
\left|\frac{V_{out}}{V_{in}}(j\omega)\right| &= \frac{g_m R_D}{\sqrt{1 + \left(\omega C_L R_D\right)^2}} \\
\frac{g_m R_D}{\sqrt{1 + \left(\omega_{-1\text{ dB}} C_L R_D\right)^2}} &= 0.9 g_m R_D \\
\omega_{-1\text{ dB}} &= 4.84 \times 10^8 \text{ rad/s} \\
f_{-1\text{ dB}} &= \frac{\omega_{-1\text{ dB}}}{2\pi} = \boxed{77.1\text{ MHz}}
\end{aligned}$$

2)



-3dB bandwidth = 1 GHz

$C_L = 2pF$

Power = 2mW

Low freq gain ?

Power = $2.5V\,I_c$, $I_c = 0.8\,mA$

Dominant Pole at the output = $\frac{1}{R_1 C_L} = 2\pi\,(1GHz)$

$R_1 = 79.58$ Ohm.

Low freq gain: $-g_m R_1 = \frac{-I_c R_1}{V_T} = \frac{(79.58)(0.8)}{26}$

$A_v\Big|_{low\,freq} = -2.45$

11.3 (a)

$$\omega_{-3\text{ dB}} = \boxed{\frac{1}{\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right) C_L}}$$

(b)

$$\omega_{-3\text{ dB}} = \boxed{\frac{1}{\left(\frac{r_{\pi 2}+R_B}{1+\beta}\right) C_L}} \approx \frac{1}{\left(\frac{1}{g_{m2}} + \frac{R_B}{1+\beta}\right) C_L}$$

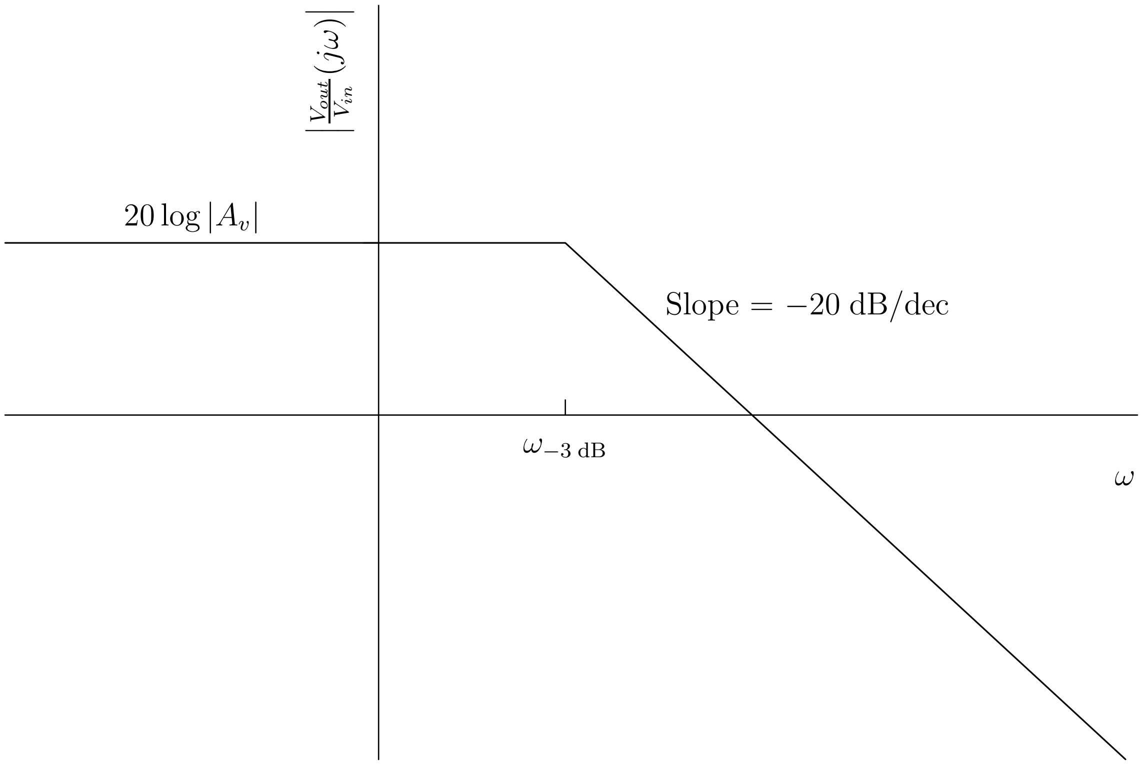
(c)

$$\omega_{-3\text{ dB}} = \boxed{\frac{1}{(r_{o1} \parallel r_{o2})\, C_L}}$$

(d)

$$\omega_{-3\text{ dB}} = \boxed{\frac{1}{\left(r_{o1} \parallel \frac{1}{g_{m2}} \parallel r_{o2}\right) C_L}}$$

11.4 Since all of these circuits are have one pole, all of the Bode plots will look qualitatively identical, with some DC gain at low frequencies that rolls off at 20 dB/dec after hitting the pole at $\omega_{-3\text{ dB}}$. This is shown in the following plot:



For each circuit, we'll derive $|A_v|$ and $\omega_{-3\text{ dB}}$, from which the Bode plot can be constructed as in the figure.

(a)

$$|A_v| = \boxed{g_{m1}\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)}$$

$$\omega_{-3\text{ dB}} = \boxed{\frac{1}{\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right) C_L}}$$

(b)

$$|A_v| = \boxed{g_{m1}\left(\frac{r_{\pi 2} + R_B}{1+\beta}\right)} \approx g_{m1}\left(\frac{1}{g_{m2}} + \frac{R_B}{1+\beta}\right)$$

$$\omega_{-3\text{ dB}} = \boxed{\frac{1}{\left(\frac{r_{\pi 2}+R_B}{1+\beta}\right) C_L}} \approx \frac{1}{\left(\frac{1}{g_{m2}} + \frac{R_B}{1+\beta}\right) C_L}$$

(c)

$$|A_v| = \boxed{g_{m1}\left(r_{o1} \parallel r_{o2}\right)}$$

$$\omega_{-3\text{ dB}} = \boxed{\frac{1}{\left(r_{o1} \parallel r_{o2}\right) C_L}}$$

(d)

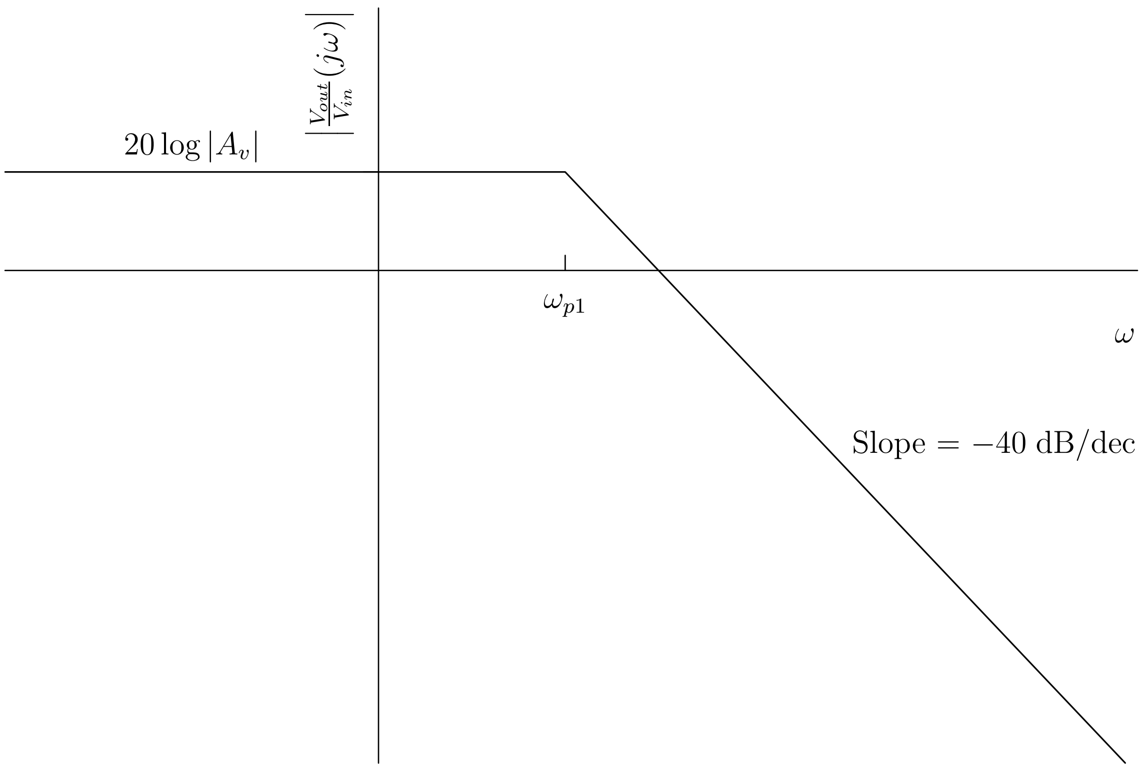
$$|A_v| = \boxed{g_{m1}\left(r_{o1} \parallel \frac{1}{g_{m2}} \parallel r_{o2}\right)}$$

$$\omega_{-3\text{ dB}} = \boxed{\frac{1}{\left(r_{o1} \parallel \frac{1}{g_{m2}} \parallel r_{o2}\right) C_L}}$$

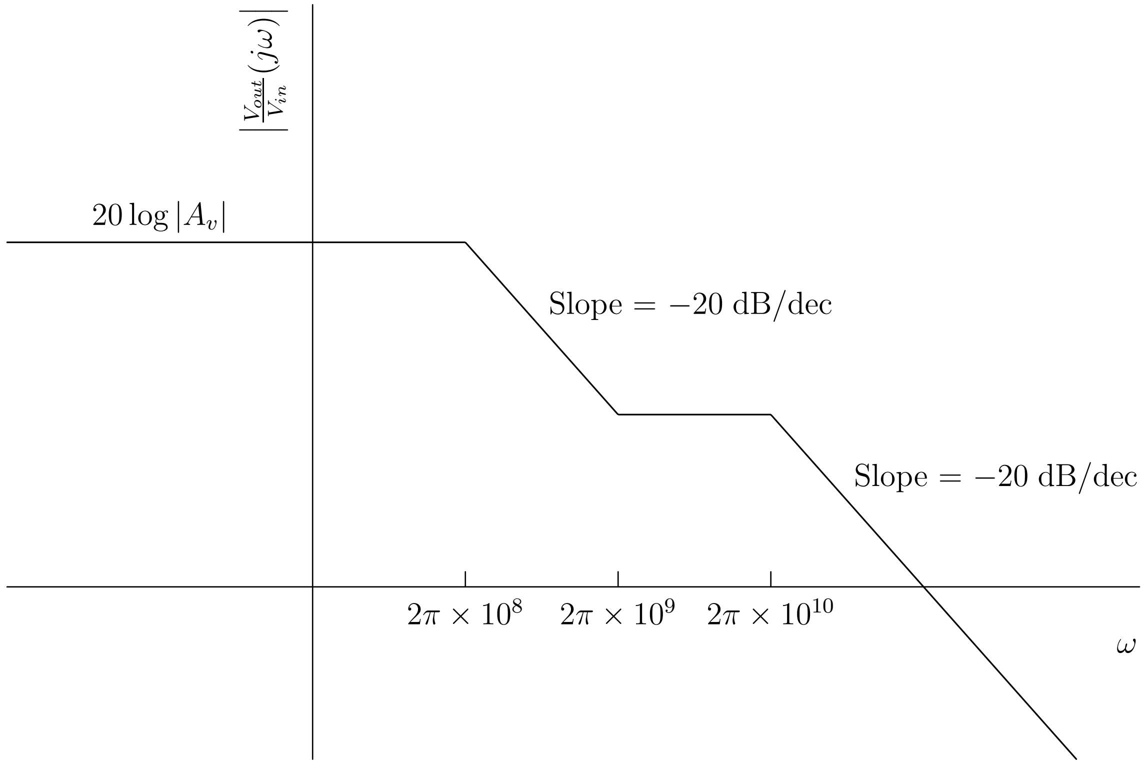
11.5 Assuming the transfer function is of the form

$$\frac{V_{out}}{V_{in}}(j\omega) = \frac{A_v}{\left(1 + j\frac{\omega}{\omega_{p1}}\right)^2}$$

we get the following Bode plot:

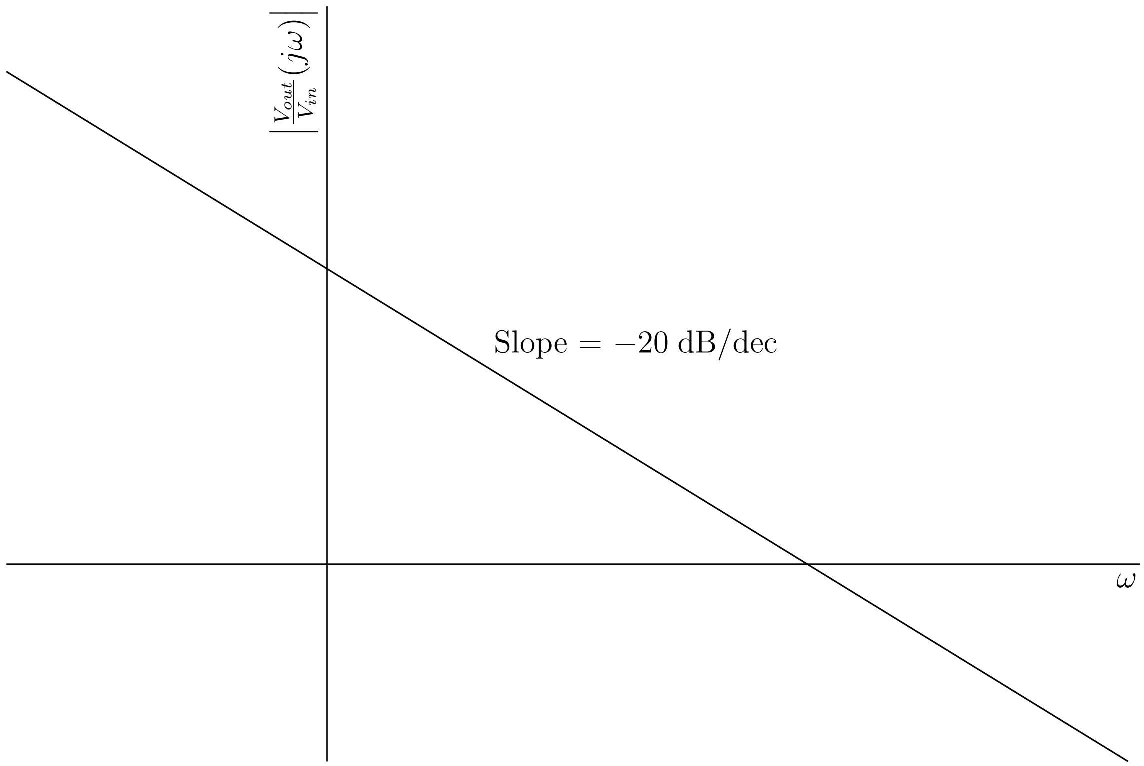


11.6

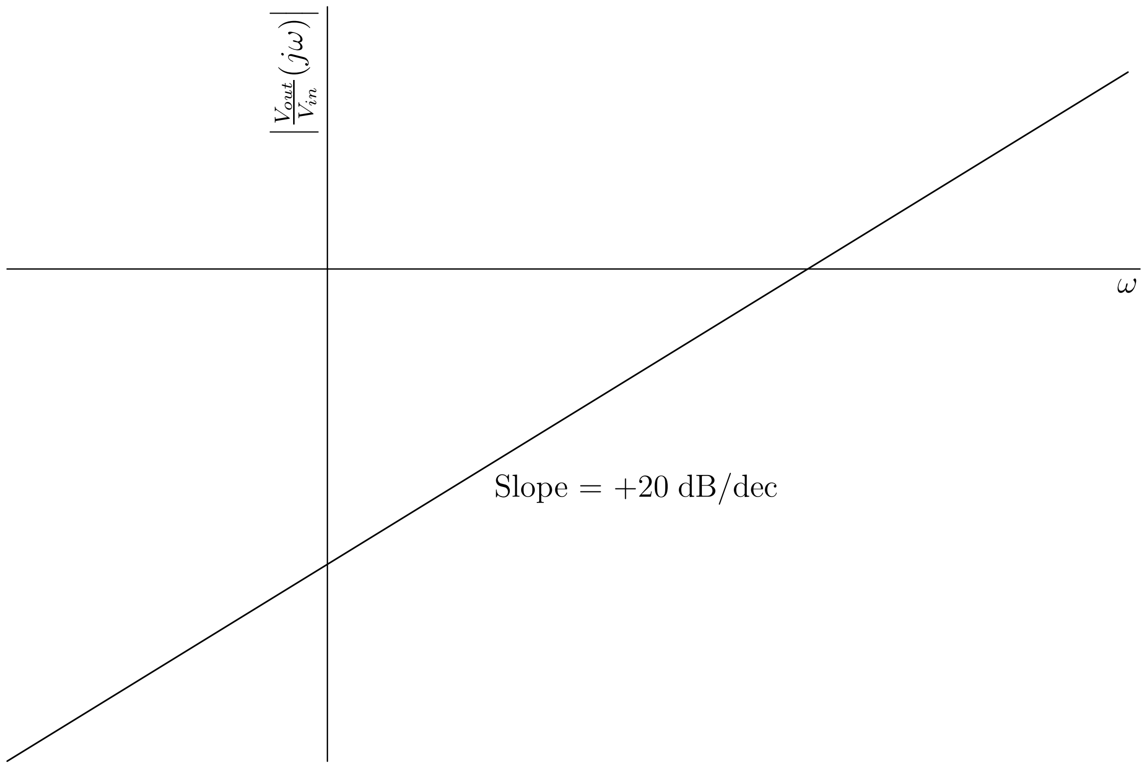


$\left|\frac{V_{out}}{V_{in}}(j\omega)\right|$
$20\log|A_v|$
Slope = −20 dB/dec
Slope = −20 dB/dec
$2\pi \times 10^8$
$2\pi \times 10^9$
$2\pi \times 10^{10}$
ω

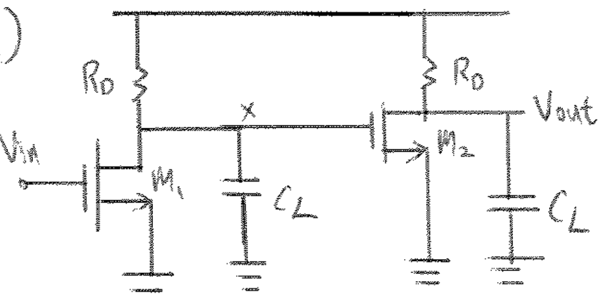
11.7 The gain at arbitrarily low frequencies approaches infinity.



11.8 The gain at arbitrarily high frequencies approaches infinity.



9)



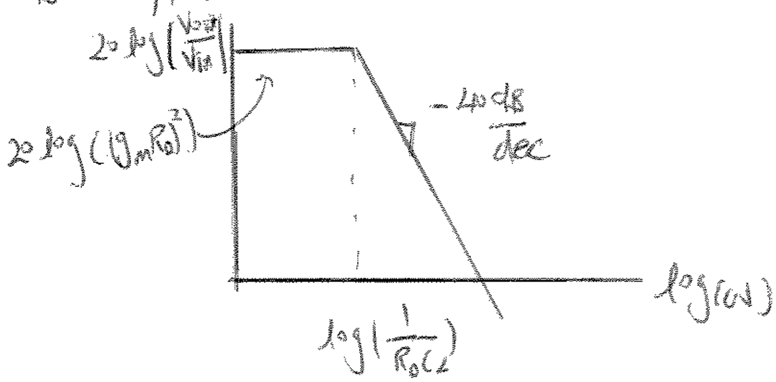
$\lambda = 0$, & neglect other caps.

DC gain: $\frac{V_x}{V_{in}} = -g_m R_D$, $\frac{V_{out}}{V_x} = -g_m R_D$

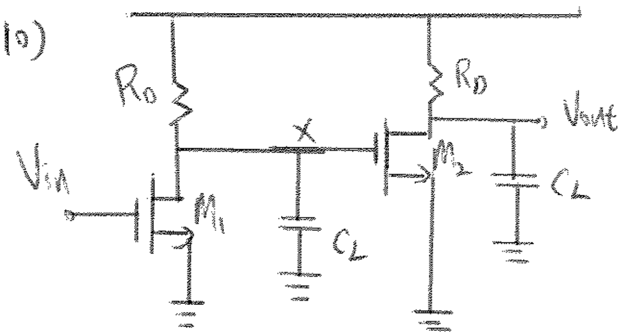
$$\frac{V_{out}}{V_{in}} = (g_m R_D)^2 \qquad \text{(At DC)}$$

2 poles at $\frac{1}{R_D C_L}$

Bode plot:



10)



$$\frac{V_x}{V_{in}}(s) = -g_m\left(R_D /\!/ \frac{1}{C_L s}\right), \quad \frac{V_{out}}{V_x}(s) = -g_m\left(\frac{R_D}{R_D C_L s+1}\right)$$

$$= -g_m\left(\frac{R_D}{R_D C_L s+1}\right)$$

$$H(s) = \frac{V_x}{V_{in}}(s)\,\frac{V_{out}}{V_x}(s) = \left(\frac{g_m R_D}{R_D C_L s+1}\right)^2$$

$$s \to \omega j, \quad H(j\omega) = \left(\frac{g_m R_D}{1 + R_D C_L j\omega}\right)^2$$

$$|H(j\omega)| = \frac{(g_m R_D)^2}{1 + (R_D C_L \omega)^2}$$

−3dB Band Width:

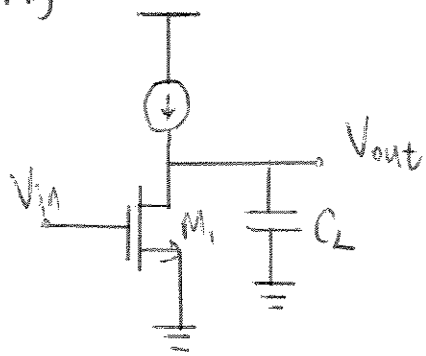
$$\frac{(g_m R_D)^2}{1 + (R_D C_L \omega)^2} = \frac{(g_m R_D)^2}{\sqrt{2}}$$

$$\Rightarrow (R_D C_L \omega)^2 + 1 = \sqrt{2}$$

$$\Rightarrow \omega = \frac{\sqrt{\sqrt{2}-1}}{R_D C_L} = \frac{0.6436}{R_D C_L} \ (\text{rad}/s)$$

$$2\pi f = \frac{0.6436}{R_D C_L} \Rightarrow f = \frac{0.10243}{R_D C_L} \ (\text{Hz})$$

1 a)



$\lambda > 0$

Since $\lambda > 0$, and we have an ideal current source, the impedance looking from out to ground is $r_o // \frac{1}{C_L s}$

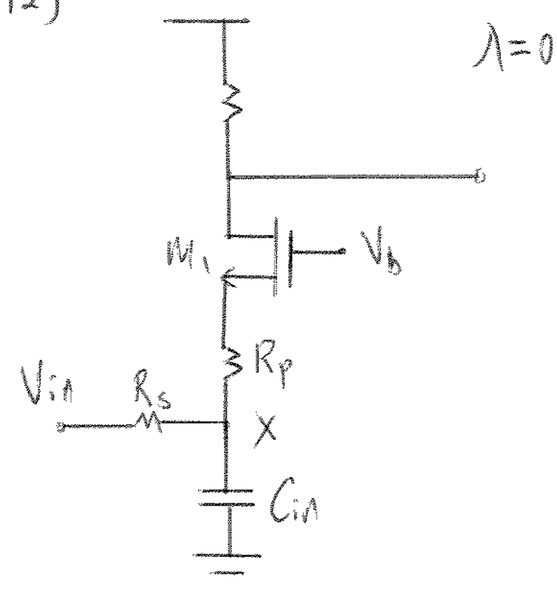
So, $V_{out} = -g_m V_{in}\left(r_o // \frac{1}{C_L s}\right)$

$$H(s) = -g_m\left(\frac{r_o}{r_o C_L s + 1}\right), \quad |H(j\omega)| = \frac{g_m r_o}{\sqrt{(r_o C_L \omega)^2 + 1}}$$

For $\lambda \to 0$, $r_o \to \infty \Rightarrow H(s) \to \frac{-g_m r_o}{r_o C_L s}$

$H(s) = \frac{-g_m}{C_L s}$. A pole at origin, thus operating as an ideal integrator.

12)



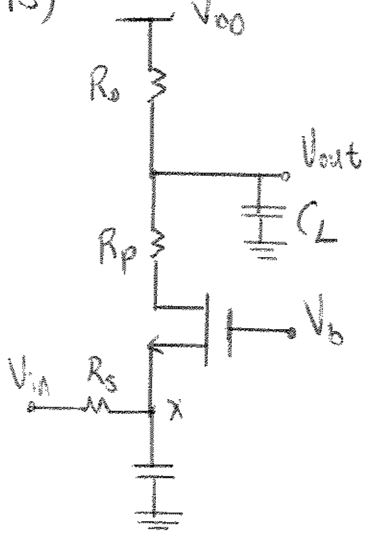
To find input pole, let $V_{in} = 0$ and find the equivalent resistance and capacitance from node X to ground.

$$R_x = R_S \,//\, \left(R_p + \frac{1}{g_{m_1}} \right), \qquad C_x = C_{in}$$

$$\omega_{pin} = \frac{1}{C_{in}\left[R_S \,//\, \left(R_p + \frac{1}{g_m}\right)\right]}$$

$$\omega_{pout} = \frac{1}{R_D C_L}$$

13)



$\lambda = 0$, neglect all other caps.

$R_X = R_S // \dfrac{1}{g_m}$

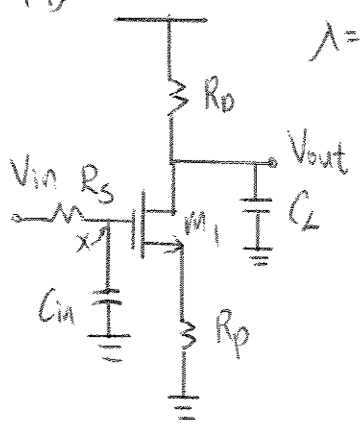
$C_X = C_{in}$

$R_{out} = R_D$ (since $r_o = \infty$)

$C_{out} = C_L$

$$\omega_{pin} = \frac{1}{(R_S // \frac{1}{g_m})\, C_{in}}, \qquad \omega_{pout} = \frac{1}{R_D C_L}$$

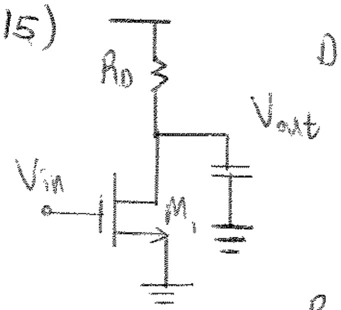
14)



$R_X = R_S$, $R_{out} \approx R_D$

$C_X = C_{in}$, $C_{out} \approx C_L$

$$\omega_{pin} = \frac{1}{R_S C_{in}} \quad , \quad \omega_{pout} = \frac{1}{R_D C_L}$$

15) 

DC Gain: $g_m R_D = \frac{2 I_D R_D}{V_{eff}}$

where $V_{eff} = V_{GS} - V_{th}$

Band Width: $\frac{1}{R_D C_L}$

Power Consumption: $V_{DD} I_D$

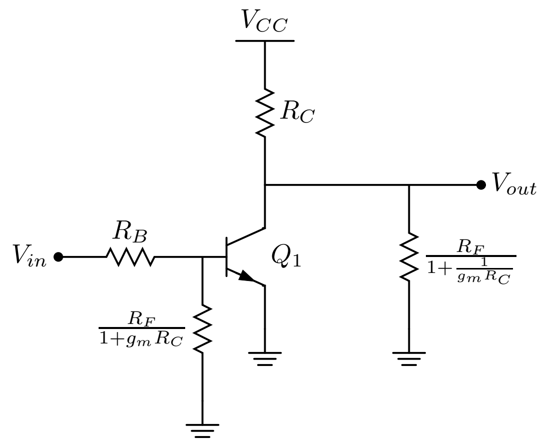
F.O.M. (11.5) $= \frac{\text{Gain} \times \text{Band Width}}{\text{Power Consumption}}$

$$= \frac{\left(\frac{2 I_D R_D}{V_{eff}}\right)\left(\frac{1}{R_D C_L}\right)}{V_{DD} I_D}$$

$$= \frac{2}{V_{eff} V_{DD} C_L}$$

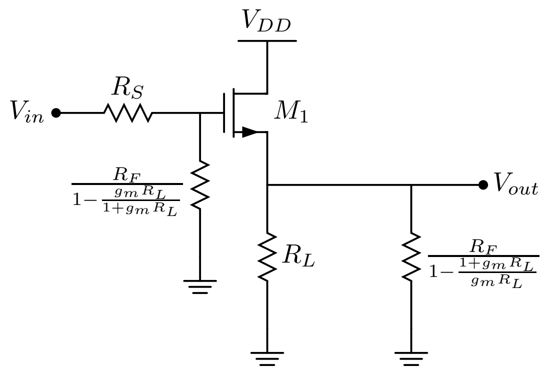
For practical design, $V_{eff} > V_t$, thus bipolar has a larger F.O.M. than MOS.

11.16 Using Miller's theorem, we can split the resistor R_F as follows:



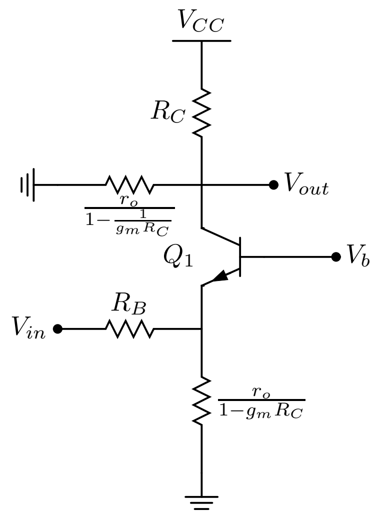
$$A_v = \boxed{-g_m \left(\frac{r_\pi \parallel \frac{R_F}{1+g_m R_C}}{R_B + r_\pi \parallel \frac{R_F}{1+g_m R_C}} \right) \left(R_C \parallel \frac{R_F}{1+\frac{1}{g_m R_C}} \right)}$$

11.17 Using Miller's theorem, we can split the resistor R_F as follows:



$$A_v = \boxed{\left(\frac{\frac{R_F}{1-\frac{g_m R_L}{1+g_m R_L}}}{R_S + \frac{R_F}{1-\frac{g_m R_L}{1+g_m R_L}}}\right)\left(\frac{g_m\left(R_L \parallel \frac{R_F}{1-\frac{1+g_m R_L}{g_m R_L}}\right)}{1+g_m\left(R_L \parallel \frac{R_F}{1-\frac{1+g_m R_L}{g_m R_L}}\right)}\right)}$$

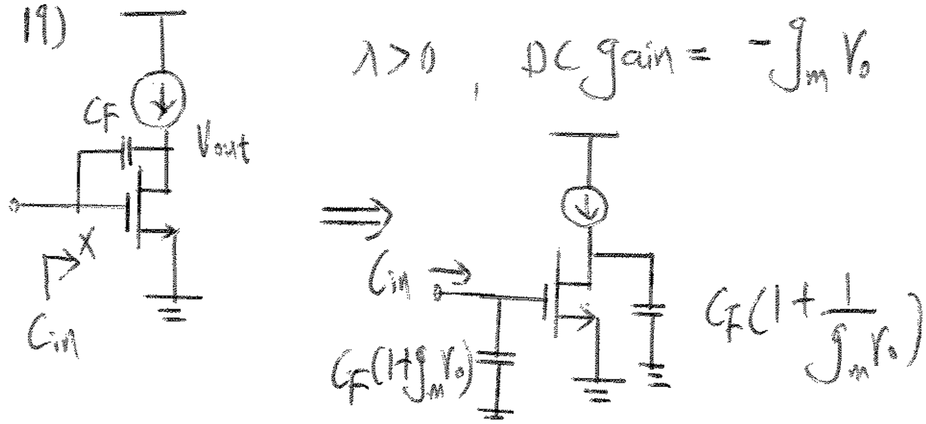
11.18 Using Miller's theorem, we can split the resistor r_o as follows:



$$A_v = \boxed{g_m \left(\frac{\frac{1}{g_m} \parallel r_\pi \parallel \frac{r_o}{1-g_m R_C}}{R_B + \frac{1}{g_m} \parallel r_\pi \parallel \frac{r_o}{1-g_m R_C}} \right) \left(R_C \parallel \frac{r_o}{1 - \frac{1}{g_m R_C}} \right)}$$

19)

$\lambda > 0$, DC gain $= -g_m r_o$

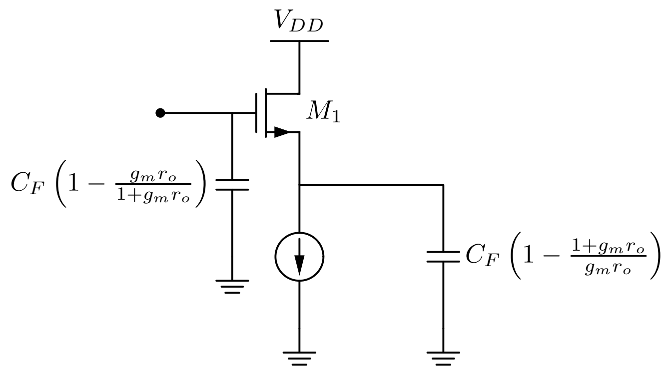


$C_{in} = C_F(1+g_m r_o)$, neglecting other caps.

As $\lambda \to 0$, $r_o \to \infty$, DC gain $\to \infty$,

$C_{in} \to \infty$, this bandwidth will $\to 0$.

11.20 Using Miller's theorem, we can split the capacitor C_F as follows (note that the DC gain is $A_v = \frac{g_m r_o}{1+g_m r_o}$):

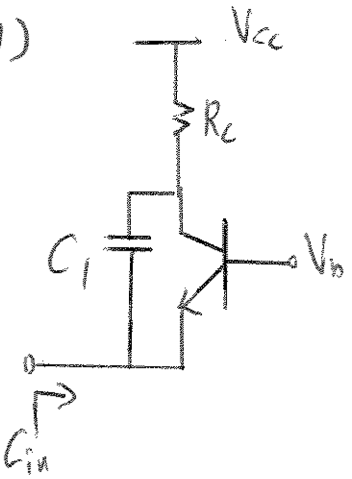


Thus, we have

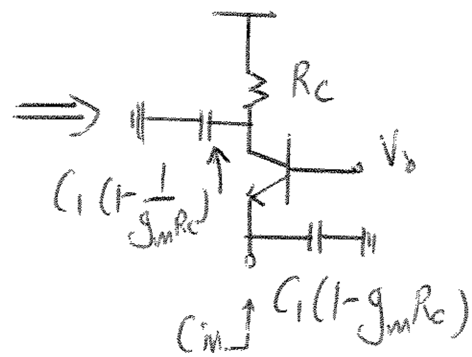
$$C_{in} = \boxed{C_F\left(1-\frac{g_m r_o}{1+g_m r_o}\right)}$$

As $\lambda \to 0$, $r_o \to \infty$, meaning the gain approaches 1. When this happens, the input capacitance goes to zero.

21)



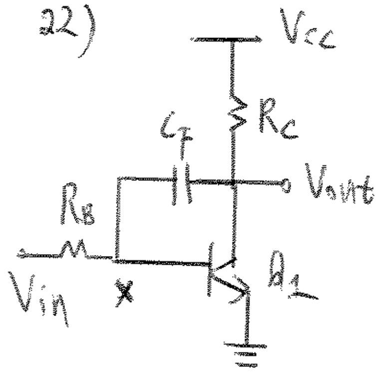
DC gain: $g_m R_c$



$$C_{in} = C_1 (1 - g_m R_c).$$

If $g_m R_c$ is designed to be larger than 1, as it normally would, we will have inductive action.

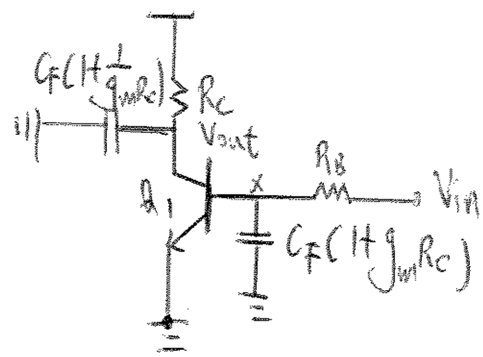
22)



DC gain (from x to out):

$-g_m R_c$

$\Downarrow$



$$C_{in} = C_F(1+g_m R_c)$$

$$R_{in} = R_B // r_\pi$$

$$C_{out} = C_F\left(1+\frac{1}{g_m R_c}\right)$$

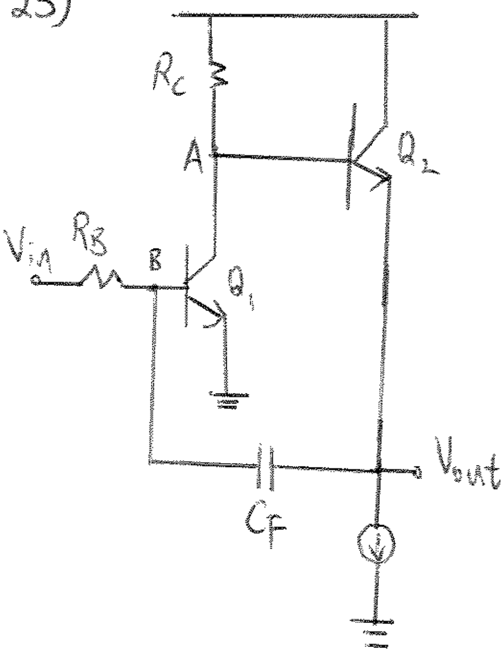
$$R_{out} = R_c$$

$$\omega_{pin} = \frac{1}{R_B // r_\pi \left[C_F(1+g_m R_c)\right]}$$

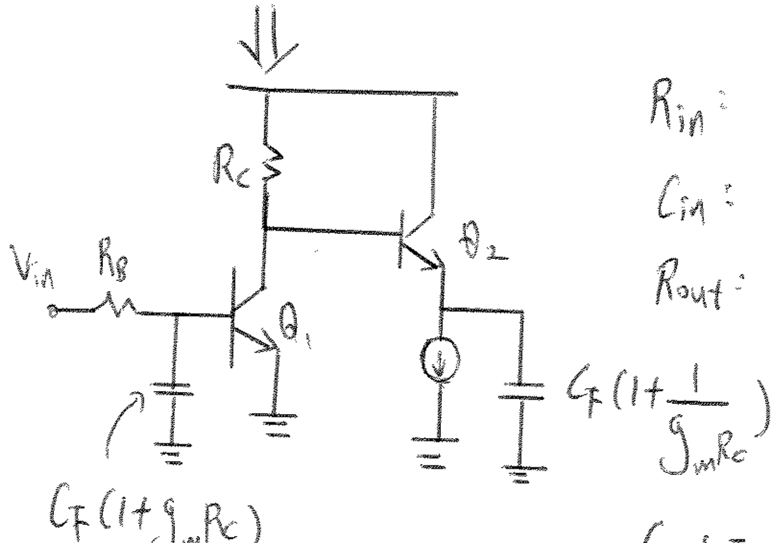
$$\omega_{pout} = \frac{1}{R_c C_F\left(1+\frac{1}{g_m R_c}\right)} \approx \frac{1}{R_c C_F}$$

(If $g_m R_c \gg 1$)

23)



The gain from B to A is $-g_m R_c$, from A to out is 1 (since we have an ideal current source). So the gain from B to out is $-g_m R_c$.



R_{in}: $R_B // r_\pi$

C_{in}: $C_F(1+g_m R_c)$

R_{out}: $\frac{1}{g_m} + \frac{R_c}{\beta+1}$

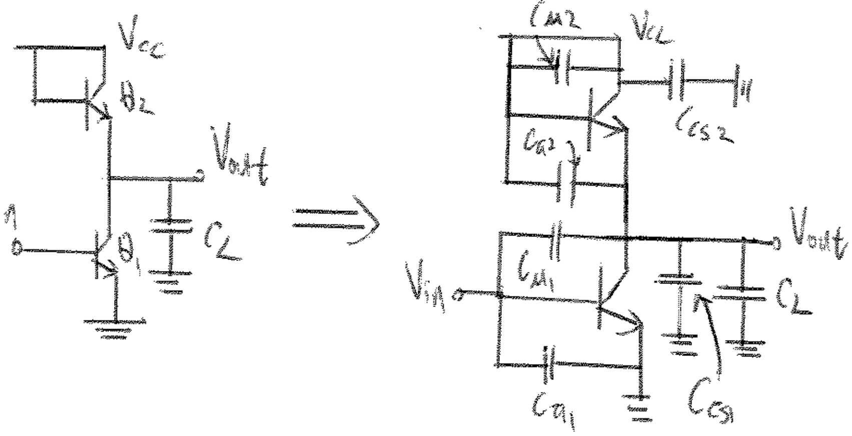
$C_{out} = C_F(1+\frac{1}{g_m R_c})$

$$\omega_{pin} = \frac{1}{R_B // r_\pi [C_F(1+g_m R_c)]}$$

$$\omega_{pout} = \frac{1}{\left(\frac{1}{g_m} + \frac{R_c}{\beta+1}\right) C_F (1+\frac{1}{g_m R_c})} \approx \frac{1}{\left(\frac{1}{g_m} + \frac{R_c}{\beta+1}\right) C_F}, \ (g_m R_c \gg 1)$$

24)

a)

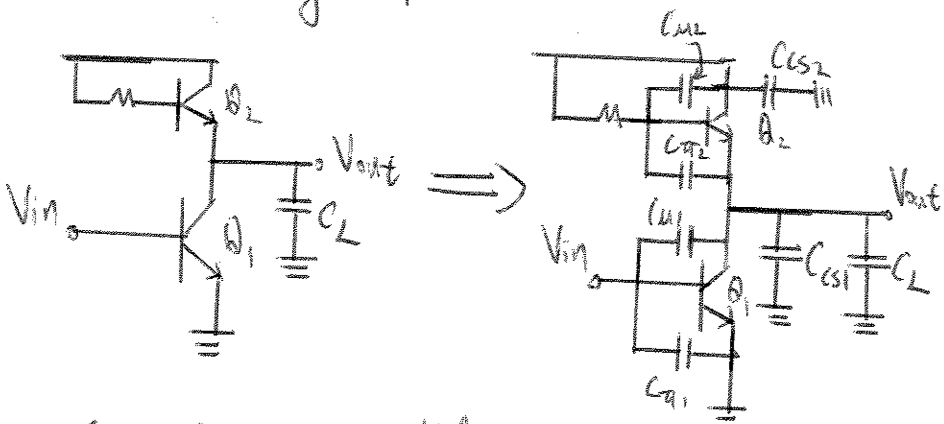


$C_{\pi 2}$, C_{CS1}, C_L are in parallel

$C_{\mu 2}$, C_{CS2} are grounded on both ends.

(and technically in parallel as well)

b)

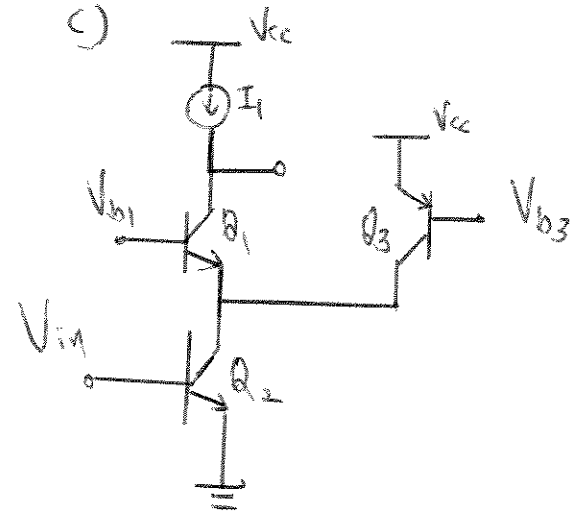


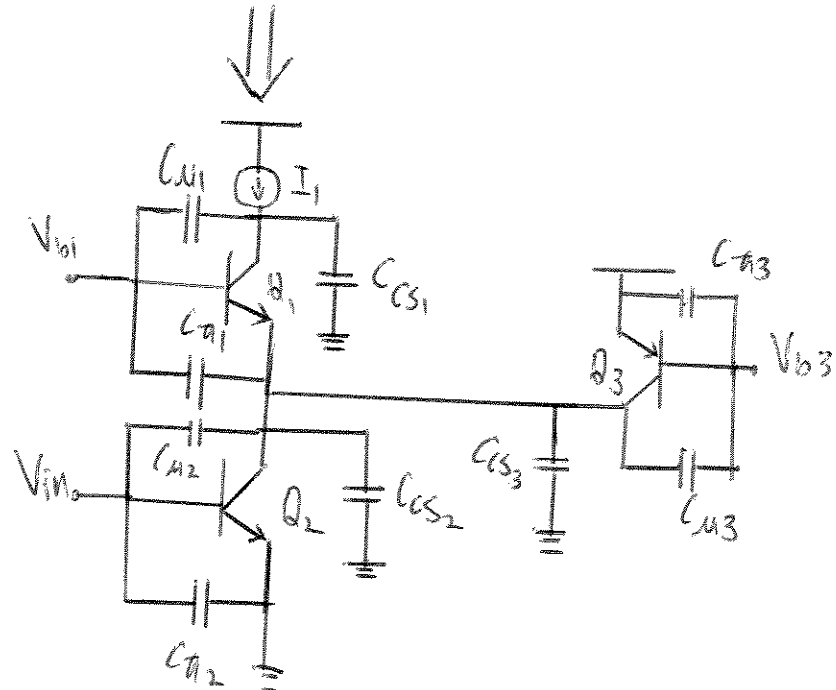
C_{CS1}, C_L are in parallel

C_{CS2} is grounded on both ends

24)

c)



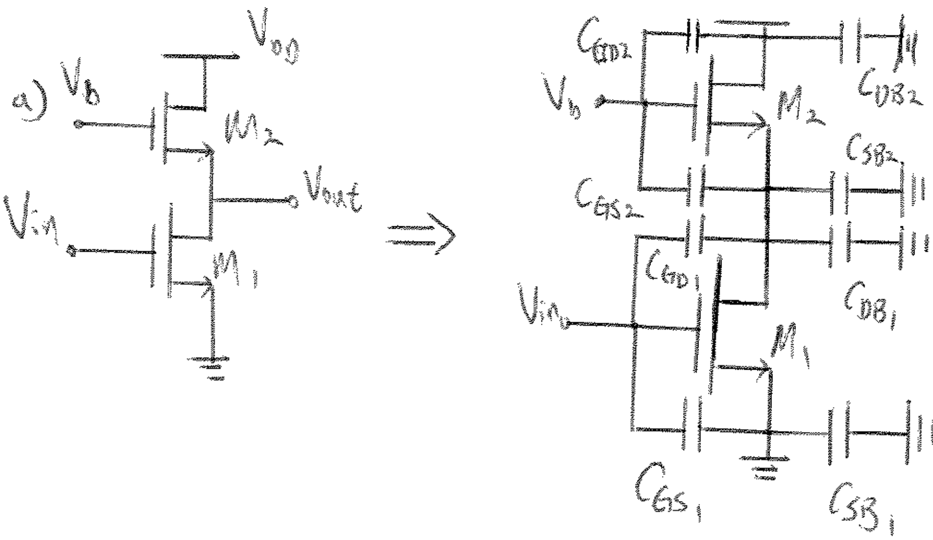


$C_{\pi 1}$, C_{cs_2}, C_{cs_3}, $C_{\mu 3}$ are in parallel
$C_{\mu 1}$, C_{cs_1} are also in parallel

$C_{\pi 3}$ is grounded on both ends

25)

a)

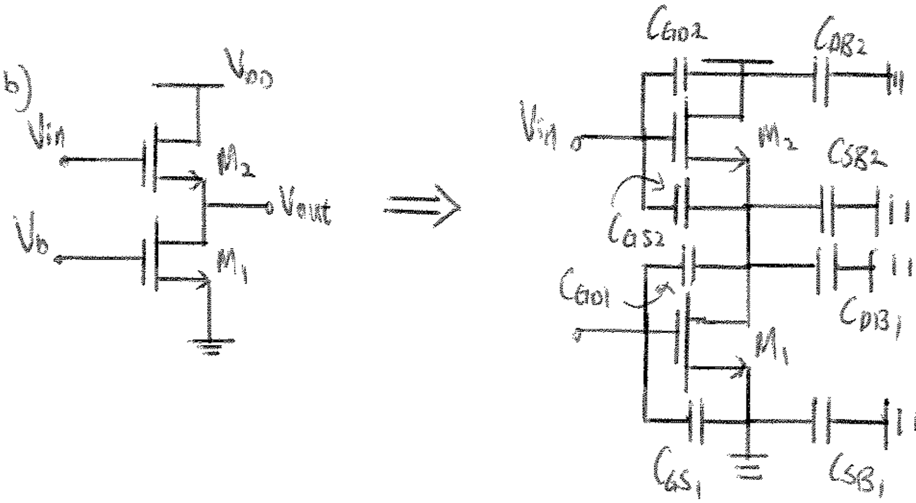


C_{GS2}, C_{SB2}, C_{DB1} are in parallel

C_{GD2}, C_{DB2} are in parallel and grounded on both ends

C_{SB1} is grounded on both ends.

b)



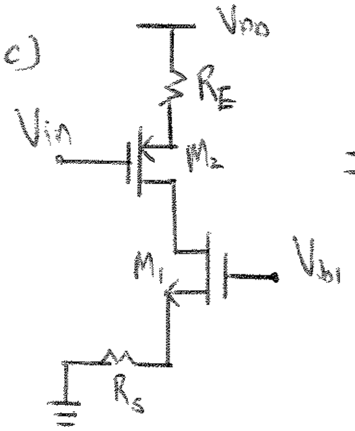
C_{GD1}, C_{DB1}, C_{SB2} are in parallel

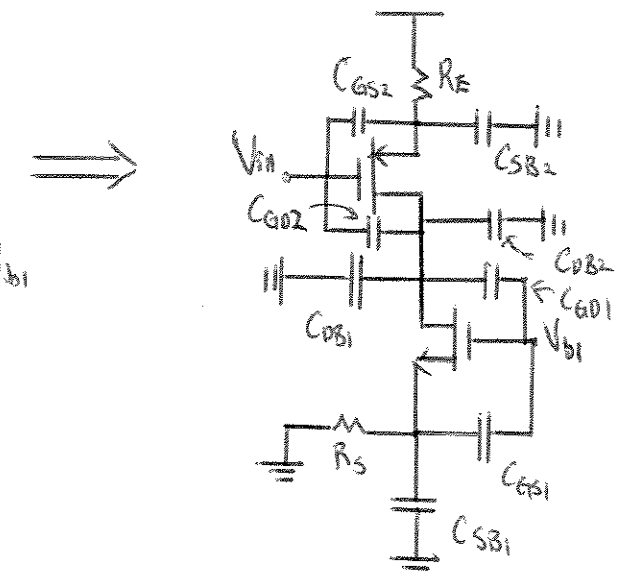
C_{GS1}, C_{SB1} are in parallel and grounded on both ends

C_{DB2} is grounded on both ends.

25)

c)

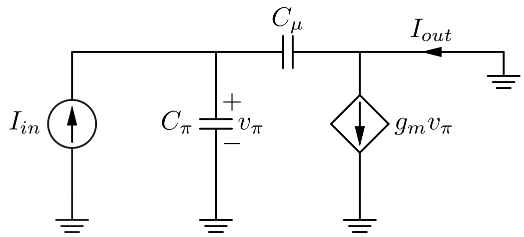




C_{DB2}, C_{GD1}, C_{DB1}, are in parallel

C_{SB1}, C_{GS1} are also in parallel.

11.26 At high frequencies (such as f_T), we can neglect the effects of r_π and r_o, since the low impedances of the capacitors will dominate at high frequencies. Thus, we can draw the following small-signal model to find f_T (for BJTs):



$$
\begin{aligned}
I_{in} &= j\omega v_\pi \left(C_\pi + C_\mu\right) \\
I_\pi &= \frac{I_{in}}{j\omega \left(C_\pi + C_\mu\right)} \\
I_{out} &= g_m v_\pi - j\omega C_\mu v_\pi \\
&= v_\pi \left(g_m - j\omega C_\mu\right) \\
&= \frac{I_{in}}{j\omega \left(C_\pi + C_\mu\right)} \left(g_m - j\omega C_\mu\right) \\
\frac{I_{out}}{I_{in}} &= \frac{g_m - j\omega C_\mu}{j\omega \left(C_\pi + C_\mu\right)} \\
\left|\frac{I_{out}}{I_{in}}\right| &= \frac{\sqrt{g_m^2 + \left(\omega C_\mu\right)^2}}{\omega \left(C_\pi + C_\mu\right)} \\
\frac{\sqrt{g_m^2 + \left(\omega_T C_\mu\right)^2}}{\omega_T \left(C_\pi + C_\mu\right)} &= 1 \\
g_m^2 + \omega_T^2 C_\mu^2 &= \omega_T^2 \left(C_\pi^2 + 2C_\pi C_\mu + C_\mu^2\right) \\
g_m^2 &= \omega_T^2 \left(C_\pi^2 + 2C_\pi C_\mu\right) \\
\omega_T &= \frac{g_m}{\sqrt{C_\pi^2 + 2C_\pi C_\mu}} \\
f_T &= \boxed{\frac{g_m}{2\pi\sqrt{C_\pi^2 + 2C_\pi C_\mu}}}
\end{aligned}
$$

The derivation of f_T for a MOSFET is identical to the derivation of f_T for a BJT, except we have C_{GS} instead of C_π and C_{GD} instead of C_μ. Thus, we have:

$$
f_T = \boxed{\frac{g_m}{2\pi\sqrt{C_{GS}^2 + 2C_{GS}C_{GD}}}}
$$

27)

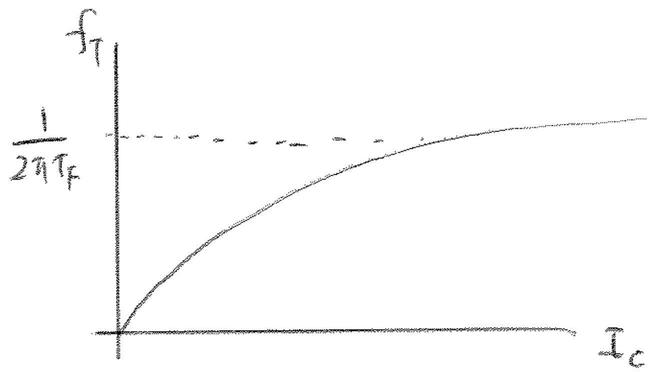
$$C_\pi = g_m \tau_F + C_{je}$$

$$2\pi f_T = \frac{g_m}{C_\pi} = \frac{g_m}{g_m \tau_F + C_{je}}$$

Assume C_{je} to be independent of I_C.

a) $$2\pi f_T = \frac{\frac{I_C}{V_T}}{\frac{I_C}{V_T}\tau_F + C_{je}} \Rightarrow f_T = \frac{I_C}{2\pi(I_C\tau_F + V_T C_{je})}$$

b)



As $I_C \to \infty$, $f_T \to \frac{1}{2\pi\tau_F}$

28)

$$C_{GS} \approx \left(\frac{2}{3}\right) W L C_{ox}$$

$$2\pi f_T = \frac{g_m}{C_{GS}} = \frac{\frac{W}{L}\mu_n C_{ox}(V_{GS}-V_{TH})}{\frac{2}{3} W L \ C_{ox}}$$

$$2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(\ V_{GS} - V_{TH})$$

29)

$$2\pi f_T = \frac{3}{2}\frac{2I_D}{WLC_{ox}}\frac{1}{(V_{GS}-V_{TH})}$$

Apparently, f_T decreases with the overdrive.

However, when we look closely, I_D is actually proportional to $(V_{GS}-V_{TH})^2$ (In Saturation), so f_T is proportional to $(V_{GS}-V_{TH})$.

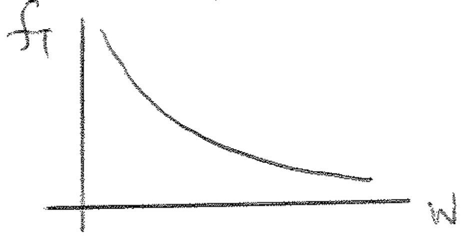
3c)

a) As $W \uparrow$, $(V_{GS} - V_{TH})$ has to $\downarrow$ by

$\frac{1}{\sqrt{W}}$ in order to maintain I_D constant

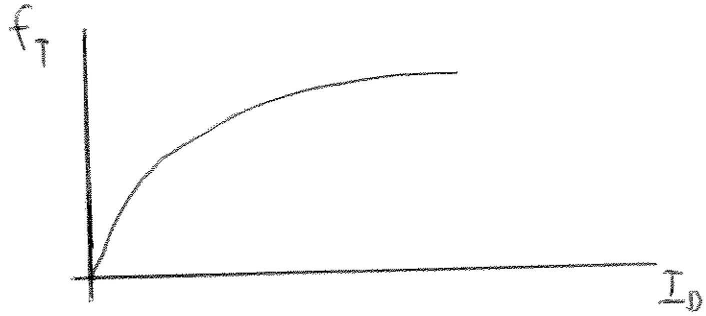
using equation $2\pi f_T = \frac{3}{2} \frac{\mu_n}{L^2} (V_{GS} - V_{TH})$

$2\pi f_T \propto \frac{1}{\sqrt{W}}$



b) $I_D \uparrow$, W constant it means $V_{GS} - V_{TH} \uparrow$ with $\sqrt{I_D}$. Using equation $2\pi f_T = \frac{3}{2} \frac{\mu_n}{L^2} (V_{GS} - V_{TH})$

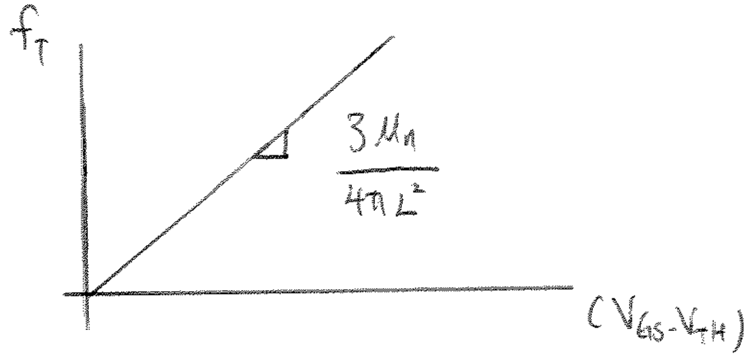
$2\pi f \propto \sqrt{I_D}$



31)

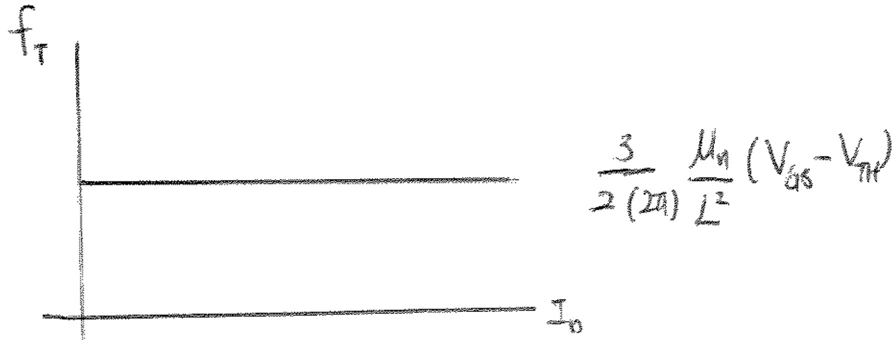
Using equation $2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS}-V_{TH})$

a) $2\pi f_T \propto (V_{GS}-V_{TH})$



b) Using equation $2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS}-V_{TH})$

$2\pi f$ = constant for all I_D.

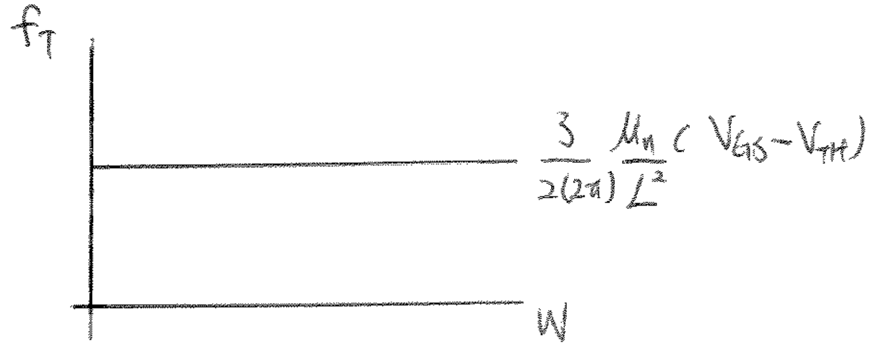


32)

a)

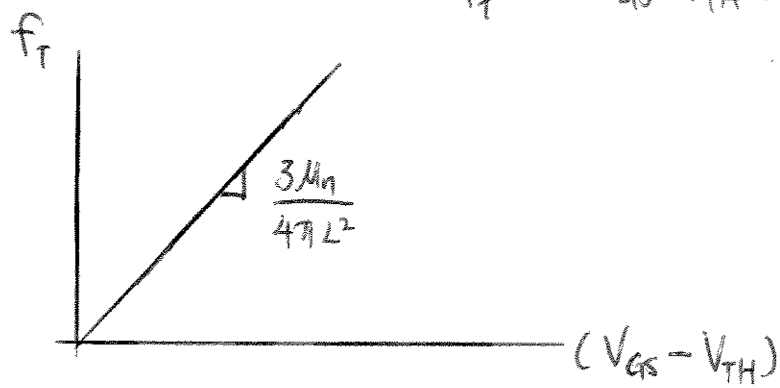
Using equation $2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS}-V_{TH})$

we know that $2\pi f_T$ is constant for all W.



b) Using equation $2\pi f_T = \frac{3}{2}\frac{\mu_n}{L^2}(V_{GS}-V_{TH})$,

we know that $2\pi f_T \propto (V_{GS}-V_{TH})$.



33)

a) $I_D = \frac{1}{2} \frac{W}{L} \mu_n C_{ox} (V_{GS} - V_{TH})^2$

As $L \uparrow$, to maintain the same current and overdrive voltage, $W \uparrow$ as well.

So W also $2X$.

b) Since $2\pi f_T = \frac{3}{2} \frac{\mu_n}{L^2} (V_{GS} - V_{TH})$, and

L $2X$ while $(V_{GS} - V_{TH})$ is constant,

$f_T \downarrow$ by $\frac{3}{4}$ or $f_{T_{new}} = \frac{1}{4} f_{T_{old}}$.

34)

a) $V_{GS} - V_{TH} \longrightarrow \frac{1}{2}(V_{GS} - V_{TH})$

Constant I_D and $W \uparrow$ (L constant)

$$2\pi f_T = \frac{3}{2} \frac{\mu_n}{L^2} (V_{GS} - V_{TH})$$

$$f_{T_{new}} = \frac{f_{T,old}}{2}$$

b) $V_{GS} - V_{TH} \longrightarrow \frac{1}{2}(V_{GS} - V_{TH})$

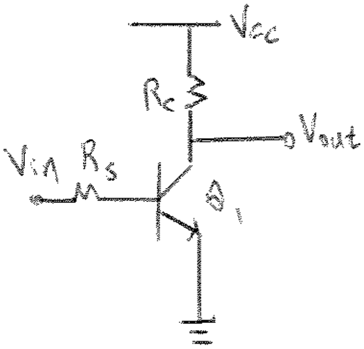
Constant W and $I_D \downarrow$ (L constant)

$$2\pi f_T = \frac{3}{2} \frac{\mu_n}{L^2} (V_{GS} - V_{TH})$$

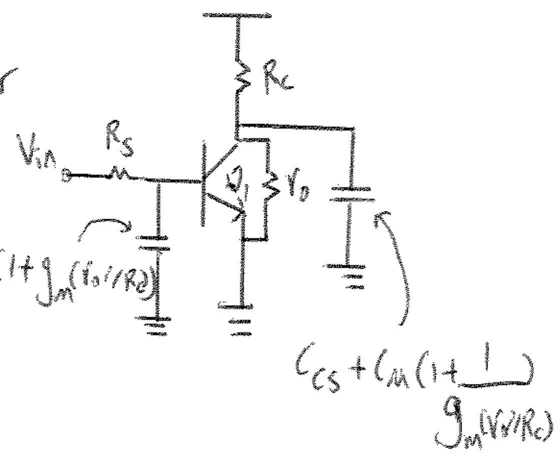
$$f_{T_{new}} = \frac{f_{T,old}}{2}$$

35)

Bipolar CE Stage



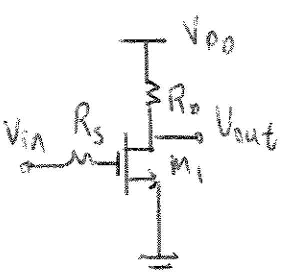
After Miller $\Longrightarrow$



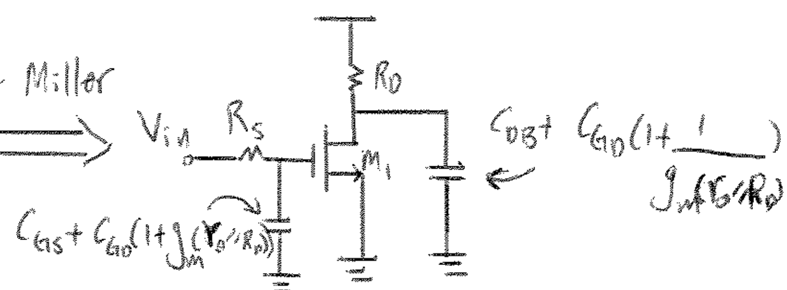
$$\omega_{p_{in}} : \frac{1}{(R_S // r_\pi)[C_\pi + C_\mu(1+g_m(r_o // R_C))]}$$

$$\omega_{p_{out}} : \frac{1}{(R_C // r_o)[C_{CS} + C_\mu(1+1/g_m(r_o // R_C))]}$$

MOS CS Stage



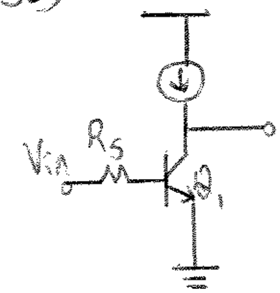
After Miller $\Longrightarrow$

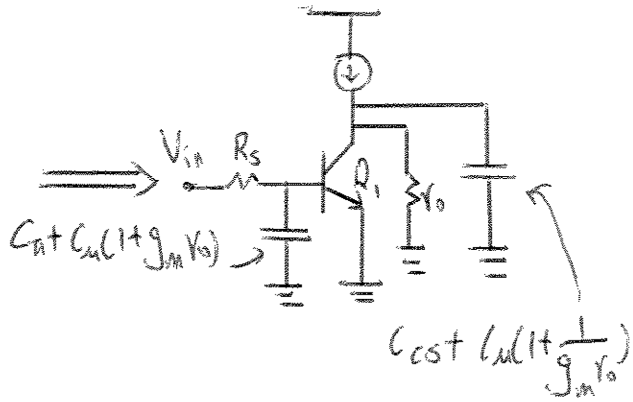


$$\omega_{p_{in}} : \frac{1}{R_S[C_{GS} + C_{GD}(1+g_m(r_o // R_D))]}$$

$$\omega_{p_{out}} : \frac{1}{(R_D // r_o)[C_{DB} + C_{GD}(1+1/g_m(r_o // R_D))]}$$

36)





$$\omega_{p\,in} = \frac{1}{(R_S \parallel r_\pi)\left[C_\pi + C_\mu(1+g_m r_o)\right]}$$

$$\omega_{p\,out} = \frac{1}{r_o\left[C_{cs} + C_\mu(1+1/g_m r_o)\right]}$$

$$H(s) = \frac{\text{DC gain}}{\left(1+\frac{s}{\omega_{p\,in}}\right)\left(1+\frac{s}{\omega_{p\,out}}\right)}$$

$$H(s) = \frac{g_m r_o\,(r_\pi/(r_\pi+R_S))}{\left(1+\frac{s}{1/(R_S \parallel r_\pi)\left[C_\pi+C_\mu(1+g_m r_o)\right]}\right)\left(1+\frac{s}{1/(r_o\left[C_{cs}+C_\mu(1+\frac{1}{g_m r_o})\right])}\right)}$$

11.37 Using Miller's theorem to split $C_{\mu 1}$, we have:

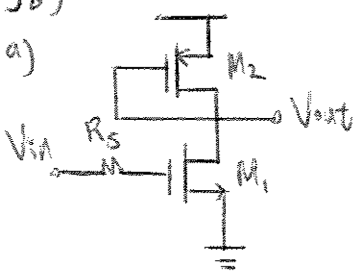
$$\omega_{p,in} = \boxed{\frac{1}{\left(R_S \parallel r_{\pi 1}\right)\left\{C_{\pi 1} + C_{\mu 1}\left[1 + g_{m1}\left(r_{o1} \parallel r_{o2}\right)\right]\right\}}}$$

$$\omega_{p,out} = \boxed{\frac{1}{\left(r_{o1} \parallel r_{o2}\right)\left\{C_{\mu 2} + C_{CS1} + C_{CS2} + C_{\mu 1}\left[1 + \frac{1}{g_{m1}\left(r_{o1} \| r_{o2}\right)}\right]\right\}}}$$

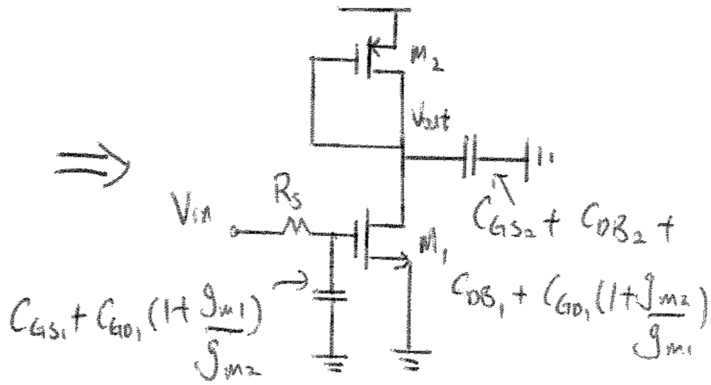
$$\frac{V_{out}}{V_{in}}(s) = \boxed{-\frac{g_{m1}\left(\frac{r_{\pi 1}}{r_{\pi 1} + R_S}\right)\left(r_{o1} \parallel r_{o2}\right)}{\left(1 + \frac{s}{\omega_{p,in}}\right)\left(1 + \frac{s}{\omega_{p,out}}\right)}}$$

38)

a)



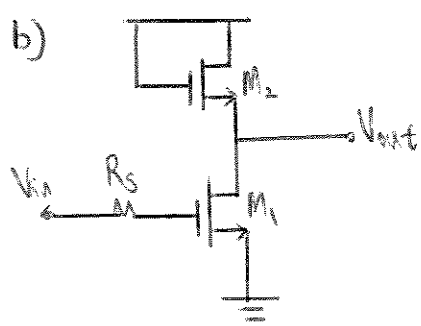
$\Longrightarrow$



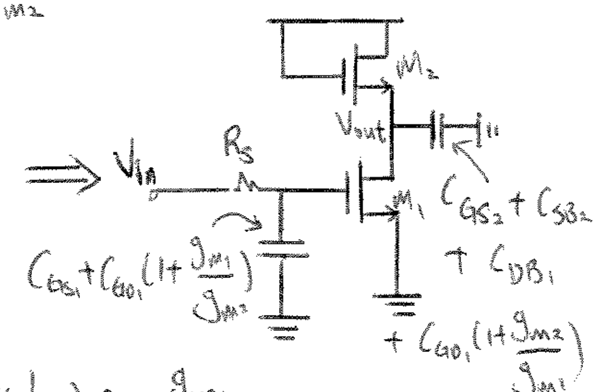
$$\text{DC gain} = -g_{m_1}\left(r_{o_1} \,//\, r_{o2} \,//\, \frac{1}{g_{m2}}\right) \approx -\frac{g_{m_1}}{g_{m_2}}$$

$$\omega_{p_{in}} = \frac{1}{R_s\left(C_{GS_1} + C_{GD_1}\left(1+\frac{g_{m1}}{g_{m2}}\right)\right)} \qquad \omega_{p_{out}} = \frac{g_{m2}}{\left(C_{GS_2} + C_{DB_2} + C_{DB_1} + C_{GD_1}\left(1+\frac{g_{m2}}{g_{m1}}\right)\right)}$$

b)



$\Longrightarrow$



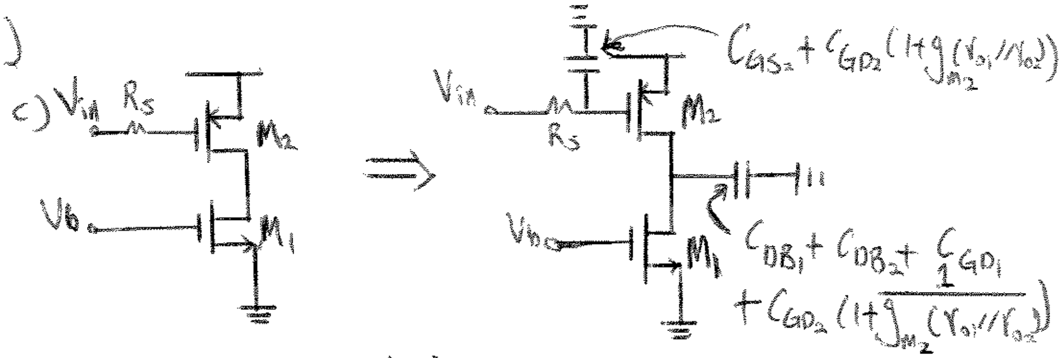
$$\text{DC gain} = -g_{m_1}\left(r_{o_1} \,//\, r_{o2} \,//\, \frac{1}{g_{m2}}\right) \approx -\frac{g_{m1}}{g_{m2}}$$

$$\omega_{p_{in}} = \frac{1}{R_s\left(C_{GS_1} + C_{GD_1}\left(1+\frac{g_{m1}}{g_{m2}}\right)\right)}$$

$$\omega_{p_{out}} = \frac{g_{m2}}{C_{SB_2} + C_{GS_2} + C_{DB_1} + C_{GD_1}\left(1+\frac{g_{m2}}{g_{m1}}\right)}$$

38)

c)



DC gain: $-g_{m2}(r_{o1} // r_{o2})$

$$\omega_{pin} = \frac{1}{R_S\left(C_{GS_2} + C_{GD_2}(1 + g_{m2}(r_{o1} // r_{o2}))\right)}$$

$$\omega_{pout} = \frac{1}{(r_{o1} // r_{o2})\left[C_{DB_1} + C_{DB_2} + C_{GD_1} + C_{GD_2}\left(1 + \frac{1}{g_{m2}(r_{o1} // r_{o2})}\right)\right]}$$

$$\omega_{pout} \approx \frac{1}{(r_{o1} // r_{o2})\left[C_{DB_1} + C_{DB_2} + C_{GD_1} + C_{GD_2}\right]}$$

Since $g_{m2}(r_{o1} // r_{o2}) >> 1$

11.39 (a)

$$\omega_{p,in} = \frac{1}{R_S\left[C_{GS} + C_{GD}\left(1 + g_m R_D\right)\right]} = \boxed{3.125 \times 10^{10}\ \text{rad/s}}$$

$$\omega_{p,out} = \frac{1}{R_D\left[C_{DB} + C_{GD}\left(1 + \frac{1}{g_m R_D}\right)\right]} = \boxed{3.846 \times 10^{10}\ \text{rad/s}}$$

(b)

$$\frac{V_{out}}{V_{Thev}}(s) = \frac{\left(C_{GD}s - g_m\right) R_D}{as^2 + bs + 1}$$

$$a = R_S R_D \left(C_{GS}C_{GD} + C_{DB}C_{GD} + C_{GS}C_{DB}\right) = 2.8 \times 10^{-22}$$

$$b = \left(1 + g_m R_D\right) C_{GD} R_S + R_S C_{GS} + R_D \left(C_{GD} + C_{DB}\right) = 5.7 \times 10^{-11}$$

Setting the denominator equal to zero and solving for s, we have:

$$s = \frac{-b \pm \sqrt{b^2 - 4a}}{2a}$$

$$|\omega_{p1}| = \boxed{1.939 \times 10^{10}\ \text{rad/s}}$$

$$|\omega_{p2}| = \boxed{1.842 \times 10^{11}\ \text{rad/s}}$$

We can see substantial differences between the poles calculated with Miller's approximation and the poles calculated from the transfer function directly. We can see that Miller's approximation does a reasonably good job of approximating the input pole (which corresponds to $|\omega_{p1}|$). However, the output pole calculated with Miller's approximation is off by nearly an order of magnitude when compared to ω_{p2}.

11.40 (a) Note that the DC gain is $A_v = -\infty$ if we assume $V_A = \infty$.

$$\omega_{p,in} = \frac{1}{(R_S \parallel r_\pi)\left[C_\pi + C_\mu (1 - A_v)\right]} = \boxed{0}$$
$$\omega_{p,out} = \boxed{0}$$

(b)

$$\frac{V_{out}}{V_{Thev}}(s) = \lim_{R_L \to \infty} \frac{(C_\mu s - g_m) R_L}{as^2 + bs + 1}$$
$$a = (R_S \parallel r_\pi) R_L (C_\pi C_\mu + C_{CS} C_\mu + C_\pi C_{CS})$$
$$b = (1 + g_m R_L) C_\mu (R_S \parallel r_\pi) + (R_S \parallel r_\pi) C_\pi + R_L (C_\mu + C_{CS})$$
$$\lim_{R_L \to \infty} \frac{(C_\mu s - g_m) R_L}{as^2 + bs + 1} = \frac{C_\mu s - g_m}{\left[(R_S \parallel r_\pi)(C_\pi C_\mu + C_{CS} C_\mu + C_\pi C_{CS})\right] s^2 + \left[g_m C_\mu (R_S \parallel r_\pi) + C_\mu + C_{CS}\right] s}$$
$$= \frac{C_\mu s - g_m}{s \left\{(R_S \parallel r_\pi)(C_\pi C_\mu + C_{CS} C_\mu + C_\pi C_{CS}) s + \left[g_m C_\mu (R_S \parallel r_\pi) + C_\mu + C_{CS}\right]\right\}}$$
$$|\omega_{p1}| = \boxed{0}$$
$$|\omega_{p2}| = \boxed{\frac{g_m C_\mu (R_S \parallel r_\pi) + C_\mu + C_{CS}}{(R_S \parallel r_\pi)(C_\pi C_\mu + C_{CS} C_\mu + C_\pi C_{CS})}}$$

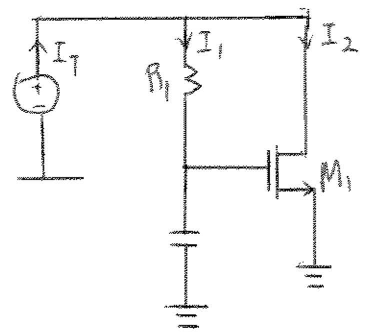
We can see that the Miller approximation correctly predicts the input pole to be at DC. However, it incorrectly estimates the output pole to be at DC as well, when in fact it is not, as we can see from the direct analysis.

11.41

$$
\begin{aligned}
|\omega_{p1}| &= \lim_{R_L \to \infty} \frac{1}{(1+g_m R_L)\, C_\mu \,(R_S \parallel r_\pi) + (R_S \parallel r_\pi)\, C_\pi + R_L\,(C_\mu + C_{CS})} = \boxed{0} \\
|\omega_{p2}| &= \lim_{R_L \to \infty} \frac{(R_S \parallel r_\pi)\, R_L\,(C_\pi C_\mu + C_{CS} C_\mu + C_\pi C_{CS})}{(1+g_m R_L)\, C_\mu \,(R_S \parallel r_\pi) + (R_S \parallel r_\pi)\, C_\pi + R_L\,(C_\mu + C_{CS})} \\
&= \boxed{\frac{(R_S \parallel r_\pi)\,(C_\pi C_\mu + C_{CS} C_\mu + C_\pi C_{CS})}{g_m C_\mu \,(R_S \parallel r_\pi) + C_\mu + C_{CS}}}
\end{aligned}
$$

The dominant-pole approximation gives the same results as analyzing the transfer function directly, as in Problem 40(b).

42)



$\lambda = 0$, and neglect other capacitances.

$$I_T = I_1 + I_2$$

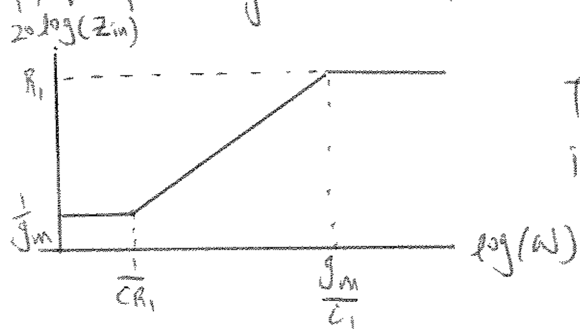
$$I_1 = \frac{V_T}{\left(R_1 + \frac{1}{C_1 S}\right)}, \quad I_2 = \frac{g_{m_1} V_T}{C_1 R_1 S + 1}$$

$$I_T = \frac{C_1 S V_T}{C_1 R_1 S + 1} + \frac{g_{m_1} V_T}{C_1 R_1 S + 1} \Rightarrow \frac{V_T}{I_T} = \frac{C_1 R_1 S + 1}{C_1 S + g_{m_1}}$$

$$S \to j\omega \Rightarrow \frac{C_1 R_1 (j\omega) + 1}{C_1 j\omega + g_{m_1}} = Z_T(j\omega)$$

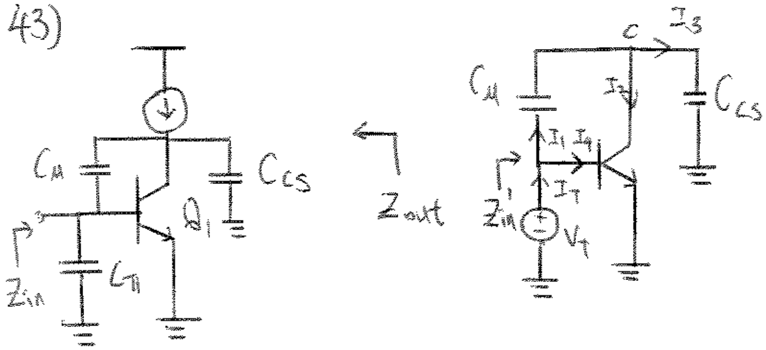
$$|Z_T| = |Z_{in}| = \frac{\sqrt{(C_1 R_1 \omega)^2 + 1}}{\sqrt{(C_1 \omega)^2 + g_{m_1}^2}} = \frac{\sqrt{(C_1 R_1 \omega)^2 + 1}}{g_m \sqrt{\left(\frac{C_1 \omega}{g_{m_1}}\right)^2 + 1}}$$

At $\omega = \frac{1}{C_1 R_1}$, we have a zero, at $\omega = \frac{g_{m_1}}{C_1}$, we have a pole. If $R_1 > \frac{1}{g_{m_1}}$, the zero is at a lower frequency than the pole, and the bode-plot for magnitude would look like the following.



The bode-plot shows an impedance that increases with frequency, an inductive behavior.

43)



$$Z_{in} = Z_{in}' \,//\, \frac{1}{C_\pi s}, \quad I_T = I_1 + I_4 = C_\mu s V_{bc} + \frac{g_m V_T}{\beta}$$

$$V_{bc} = V_T - V_c, \quad V_c = (I_1 - g_m V_T)\frac{1}{C_{cs} s}$$

$$I_1 = \left[V_T - (I_1 - g_m V_T)\frac{1}{C_{cs} s}\right] C_\mu s$$

$$I_1 = V_T\left[C_\mu s + \frac{g_m C_\mu}{C_{cs}}\right] \Big/ \left(1 + \frac{C_\mu}{C_{cs}}\right)$$

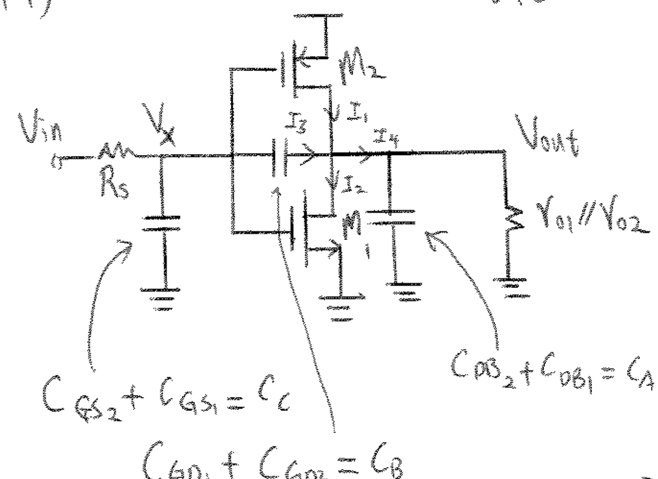
$$I_T = V_T\left[C_\mu s + \frac{g_m C_\mu}{C_{cs}}\right] \Big/ \left(1 + \frac{C_\mu}{C_{cs}}\right) + \frac{g_m V_T}{\beta}$$

$$Z_{in}' = \frac{V_T}{I_T} = \frac{1}{\dfrac{g_m}{\beta} + \dfrac{C_\mu s}{\left(1+\frac{C_\mu}{C_{cs}}\right)} + \dfrac{g_m \frac{C_\mu}{C_{cs}}}{\left(1+\frac{C_\mu}{C_{cs}}\right)}}$$

$$Z_{in} = Z_{in}' \,//\, \frac{1}{C_\pi s} = r_\pi \,//\, \frac{1}{\dfrac{C_{cs} C_\mu s}{C_{cs}+C_\mu}} \,//\, \frac{1}{C_\pi s} \,//\, \frac{C_{cs}+C_\mu}{g_m C_\mu}$$

$$Z_{out} = \frac{1}{(C_\mu + C_{cs})s}$$

44) $\lambda > 0$



$$V_{out} = I_4 \Big(\underbrace{r_{o1} // r_{o2} // \frac{1}{[C_{DB_2} + C_{DB_1}]s}}_{Z_{out}} \Big)$$

$$I_4 = I_1 + I_3 - I_2$$

$$I_1 = (0 - V_x) g_{m_2}$$

$$I_2 = V_x g_{m_1}$$

$$I_3 = (V_x - V_{out}) \left((C_{GD_1} + C_{GD_2}) s \right.$$

$$I_4 = -V_x g_{m_2} + (V_x - V_{out}) C_B s - V_x g_{m_1}$$

$$V_{out} = Z_{out} \left[-V_x (g_{m_2} + g_{m_1}) + (V_x - V_{out}) C_B s \right]$$

Writing a node equation at X.

$$\frac{V_x - V_{in}}{R_s} + V_x C_c s + (V_x - V_{out}) C_B s = 0$$

$$V_x = \frac{V_{out} C_B s + V_{in}/R_s}{(1/R_s + C_c s + C_B s)}$$

44)

Subsitute everything and we get

$$V_{out} = Z_{out}\left[-(g_{m_1}+g_{m_2})\left(\frac{V_{out}C_BS + V_{in}/R_s}{1/R_s + C_cS + C_BS}\right) + \left(\frac{V_{out}C_BS + V_{in}/R_s}{1/R_s + C_cS + C_BS} - V_{out}\right)C_BS\right]$$

collect all the Vout's on one-side and likewise for Vin's, we will get

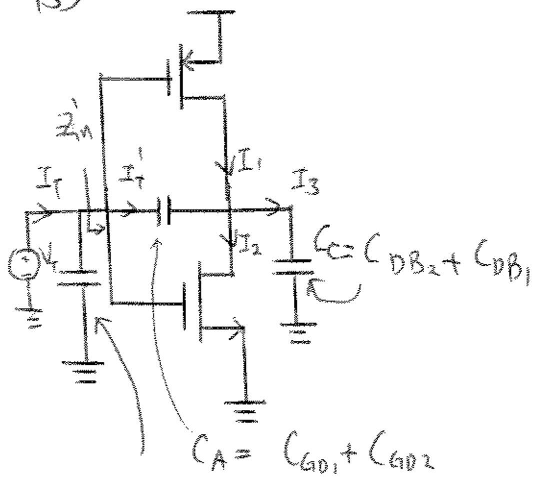
$$\frac{V_{out}}{V_{in}} = \frac{\frac{Z_{out}}{R_s}\left(C_BS - (g_{m_1}+g_{m_2})\right)}{1/R_s + (C_c+C_B)S + Z_{out}C_BS(g_{m_1}+g_{m_2}) + Z_{out}C_BS\left(\frac{1}{R_s}+(C_c+C_B)S\right) - Z_{out}C_B^2S^2}$$

where $Z_{out} = r_{o1} // r_{o2} // \dfrac{1}{[C_{DB_1}+C_{DB2}]S}$

$$C_B = C_{GD_1} + C_{GD2}$$

$$C_c = C_{GS_1} + C_{GS2}$$

45)



$C_B = C_{GS_1} + C_{GS_2}$

$$Z_{in} = \frac{V_T}{I_T} = \frac{1}{C_B} \,//\, Z_{in}'$$

$$Z_{in}' = \frac{V_T}{I_T'}$$

$$I_T' = \left[V_T - \left(I_3 \frac{1}{C_C s}\right)\right] C_A s$$

$$I_3 = I_T' - V_T g_{m2} - g_{m1} V_T$$

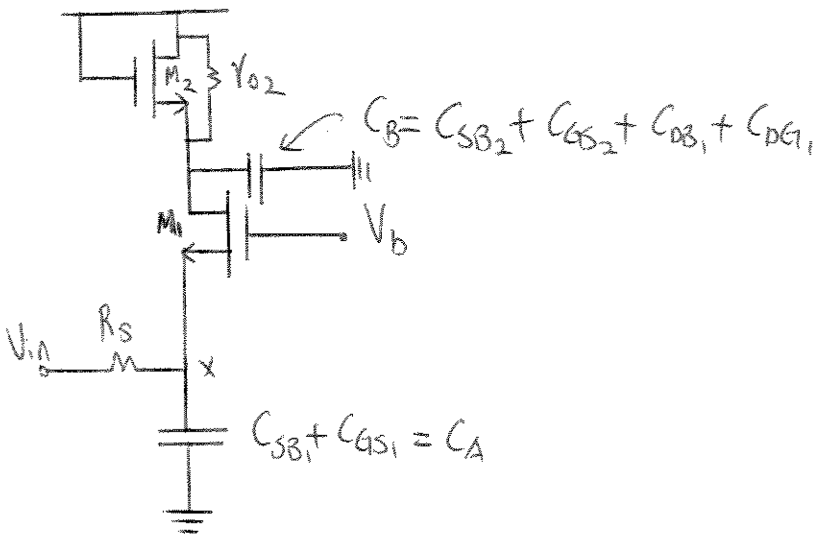
We get $\Rightarrow$ $I_T'\left(1 + \frac{C_A}{C_C}\right) = V_T\left[C_A s + (g_{m1} + g_{m2})\frac{C_A}{C_C}\right]$

$$Z_{in}' = \frac{V_T}{I_T'} = \frac{\left(1 + \frac{C_A}{C_C}\right)}{\left[C_A s + (g_{m1} + g_{m2})\frac{C_A}{C_C}\right]}$$

$$Z_{in} = \frac{1}{[C_{GS_1} + C_{GS_2}]s} \,//\, \frac{\left(1 + \frac{C_{GD_1} + C_{GD_2}}{C_{DB_1} + C_{DB_2}}\right)}{\left[(C_{GD_1} + C_{GD_2})s + (g_{m1} + g_{m2})\frac{C_{GD_1} + C_{GD_2}}{C_{DB_2} + C_{DB_1}}\right]}$$

4b)

a)



$$V_{out} = -(0 - V_x)\, g_{m_1}\left[\frac{1}{g_{m_2}} \,/\!/\, \frac{1}{C_B s}\right] = V_x\, g_{m_1}\left[\frac{1}{g_{m_2}} \,/\!/\, \frac{1}{C_B s}\right]$$

Node equation at X, $\dfrac{V_x - V_{in}}{R_s} + V_x C_A s - g_m(0 - V_x) = 0$

$$V_x\left(\frac{1}{R_s} + C_A s + g_m\right) = \frac{V_{in}}{R_s} \Rightarrow V_x = \frac{V_{in}}{(1 + R_s C_A s + R_s g_m)}$$

subsitute in V_x and solving for V_{out}/V_{in} $\Rightarrow$

$$\frac{V_{out}}{V_{in}} = \frac{g_{m_1}\left[\,1/g_{m_2} \,/\!/\, \frac{1}{C_B s}\right]}{(1 + R_s C_A s + R_s g_m)}$$

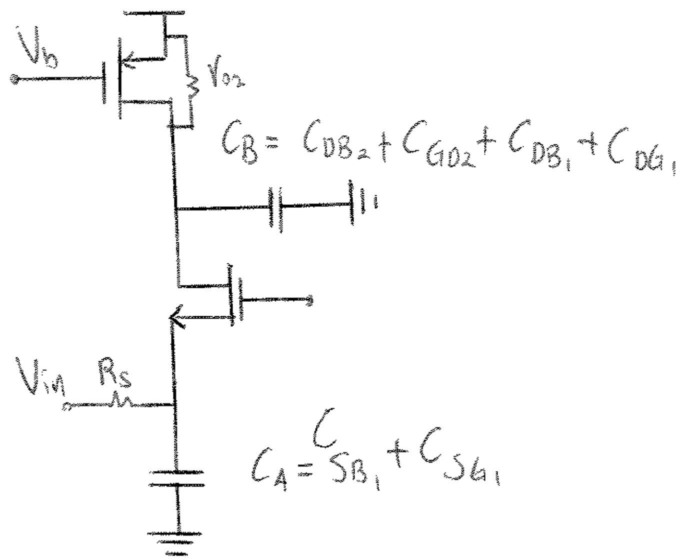
$$\frac{V_{out}}{V_{in}} = \frac{g_{m_1}(1/g_{m_2})}{(C_B(1/g_{m_2})s + 1)(1 + R_s C_A s + R_s g_{m_1})}$$

Where $C_B = C_{SB_2} + C_{GS_2} + C_{DB_1} + C_{DG_1}$

$C_A = C_{SB_1} + C_{GS_1}$

4b)

b)



Similar to part a), with $\frac{1}{g_{m_2}}$ replaced by r_{o2},

and different C_B

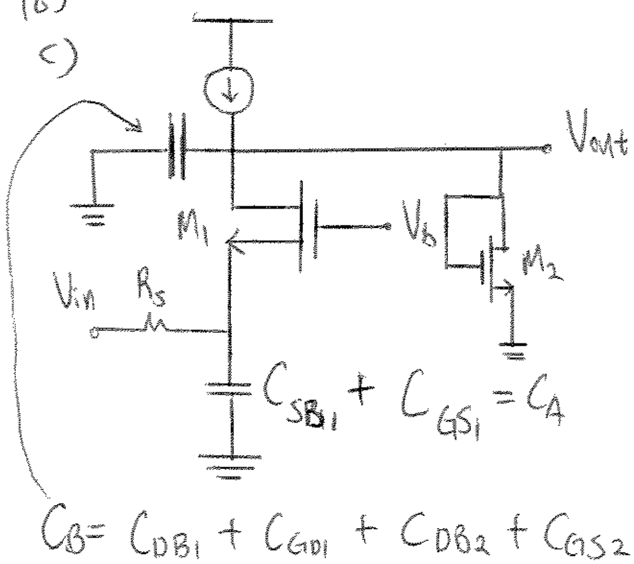
So $$\frac{V_{out}}{V_{in}} = \frac{g_{m_1} r_{o2}}{(C_B r_o S + 1)(1 + R_S C_A S + R_S g_{m_1})}$$

Where $C_B = C_{DB_2} + C_{GD2} + C_{DB_1} + C_{DG_1}$

$C_A = C_{SB_1} + C_{SG_1}$

4b)

c)



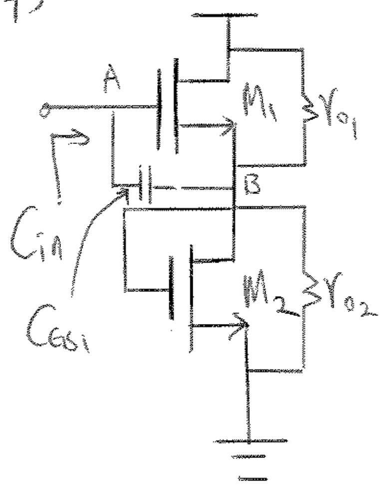
AC-wise, this circuit is very similar to part a). Its transfer function is the same as part a), except for C_B.

$$\frac{V_{out}}{V_{in}} = \frac{g_{m_1}(1/g_{m_2})}{(C_B(1/g_{m_2})S+1)(1+R_S C_A S + R_S g_{m_1})}$$

where $C_B = C_{DB_1} + C_{GD_1} + C_{DB_2} + C_{GS_2}$

$C_A = C_{SB_1} + C_{GS_1}$

47)



DC gain from A to B:

$$A_V = \frac{\frac{1}{g_{m2}} \parallel r_{o1} \parallel r_{o2}}{\frac{1}{g_{m2}} \parallel r_{o1} \parallel r_{o2} + \frac{1}{g_{m1}}}$$

$$A_V \approx \frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m2}} + \frac{1}{g_{m1}}} = \frac{g_{m1}}{g_{m1} + g_{m2}}$$

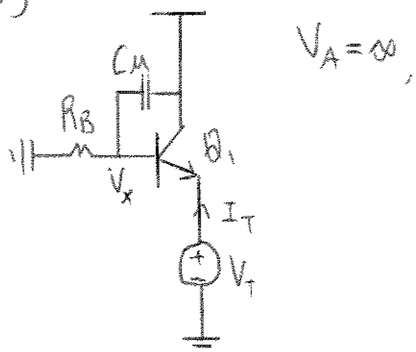
since $g_m r_o >> 1$

Using Miller's Capacitance:

$$C_{in} = C_{GS1}(1 - A_V) = C_{GS1}\left(1 - \frac{g_{m1}}{g_{m1} + g_{m2}}\right)$$

$$C_{in} = C_{GS1}\left(\frac{g_{m2}}{g_{m2} + g_{m1}}\right)$$

48)



$V_A = \infty$,

$$\frac{\beta}{\beta+1} \approx 1 \text{, if } \beta \gg 1$$

$$I_T = -(V_x - V_T)\, g_m \alpha \approx -(V_x - V_T)\, g_m$$

$$V_x = \frac{I_T}{\beta}\left(R_B /\!/ \frac{1}{C_\mu s}\right)$$

$$I_T = \left(V_T - \frac{I_T}{\beta}\left(R_B /\!/ \frac{1}{C_\mu s}\right)\right) g_m$$

$$I_T = g_m V_T - \frac{g_m}{\beta}\left(R_B /\!/ \frac{1}{C_\mu s}\right) I_T$$

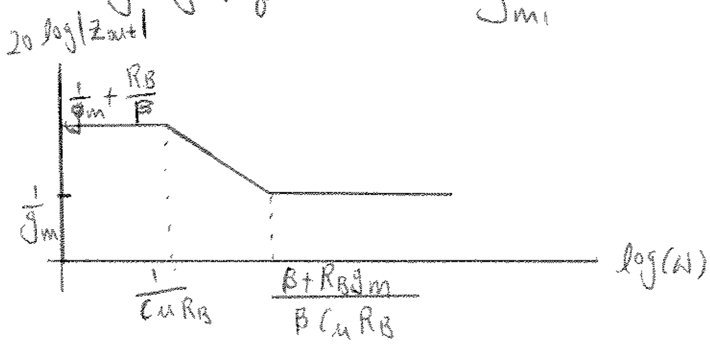
$$I_T\left(1 + \frac{g_m}{\beta}\left(R_B /\!/ \frac{1}{C_\mu s}\right)\right) = g_m V_T$$

$$\frac{V_T}{I_T} = \frac{1}{g_m} + \frac{R_B /\!/ \frac{1}{C_\mu s}}{\beta} = \frac{\beta C_\mu R_B\left(s + \frac{\beta + R_B g_m}{\beta C_\mu R_B}\right)}{g_m \beta (1 + C_\mu R_B s)}$$

Zero: $\dfrac{\beta + R_B g_m}{\beta C_\mu R_B}$, Pole: $\dfrac{1}{C_\mu R_B}$

At DC, $|Z_{out}| = \dfrac{1}{g_{m1}} + \dfrac{R_B}{\beta}$

At very high freq : $|Z_{out}| = \dfrac{1}{g_{m1}}$

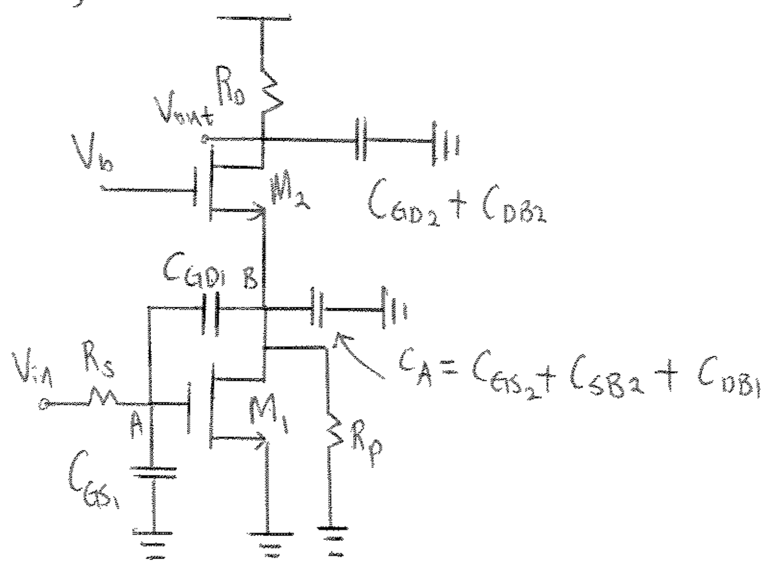


11.49

$$
\begin{aligned}
\omega_{p1} &= \frac{1}{(R_B \parallel r_{\pi 1})\left\{C_{\pi 1} + C_{\mu 1}\left[1 + g_{m1}\left(\frac{1}{g_{m2}} \parallel r_{\pi 2}\right)\right]\right\}} \\
&\approx \frac{1}{(R_B \parallel r_{\pi 1})\left\{C_{\pi 1} + C_{\mu 1}\left[1 + \frac{g_{m1}}{g_{m2}}\right]\right\}} \\
I_{C1} &= 4I_{C2} \Rightarrow g_{m1} = 4g_{m2} \\
\omega_{p1} &= \boxed{\frac{1}{(R_B \parallel r_{\pi 1})\left(C_{\pi 1} + 5C_{\mu 1}\right)}} \\
\omega_{p2} &\approx \frac{1}{\frac{1}{g_{m2}}\left[C_{CS1} + C_{CS3} + C_{\mu 3} + C_{\pi 2} + C_{\mu 1}\left(1 + \frac{g_{m2}}{g_{m1}}\right)\right]} \\
&= \boxed{\frac{g_{m2}}{C_{CS1} + C_{CS3} + C_{\mu 3} + C_{\pi 2} + \frac{5}{4}C_{\mu 1}}} \\
\omega_{p3} &= \boxed{\frac{1}{R_C\left(C_{CS2} + C_{\mu 2}\right)}}
\end{aligned}
$$

Miller's effect is more significant here than in a standard cascode. This is because the gain in the common-emitter stage is increased to four in this topology, where it is about one in a standard cascode. This means that the capacitor $C_{\mu 1}$ will be multiplied by a larger factor when using Miller's theorem.

50)



DC gain from A to B is $-g_{m_1}(R_p // \frac{1}{g_{m_2}})$

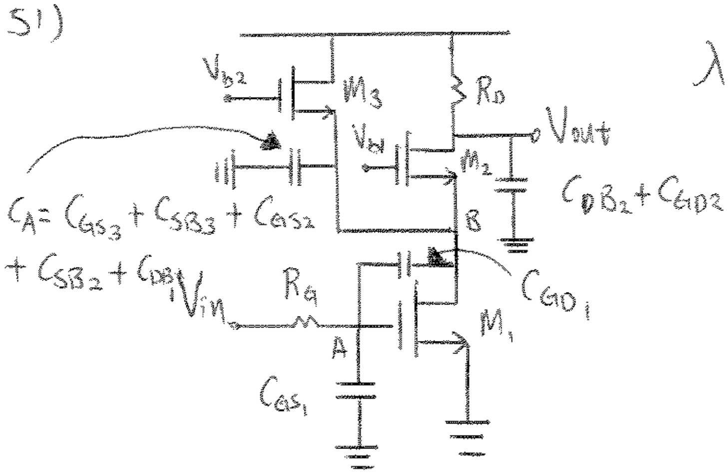
Applying Miller's Theorem:

$$\omega_{p_{in}}(\omega_{p_A}) = \frac{1}{R_S(C_{GS_1} + C_{GD_1}(1 + g_{m_1}(R_p // \frac{1}{g_{m_2}})))}$$

$$\omega_{p_B} = \frac{1}{R_p // \frac{1}{g_{m_2}} [C_{GS_2} + C_{SB_2} + C_{DB_1} + C_{GD_1}(1 + 1/g_{m_1}(R_p // \frac{1}{g_{m_2}}))]}$$

$$\omega_{p_{out}} = \frac{1}{R_D(C_{GD_2} + C_{DB_2})}$$

5')



$\lambda = 0$

DC gain from A to B: $-g_{m1}\left(\frac{1}{g_{m3}} \parallel \frac{1}{g_{m2}}\right) = -g_{m1}\left(\frac{1}{g_{m2}+g_{m3}}\right)$

Applying Miller's Theorem:

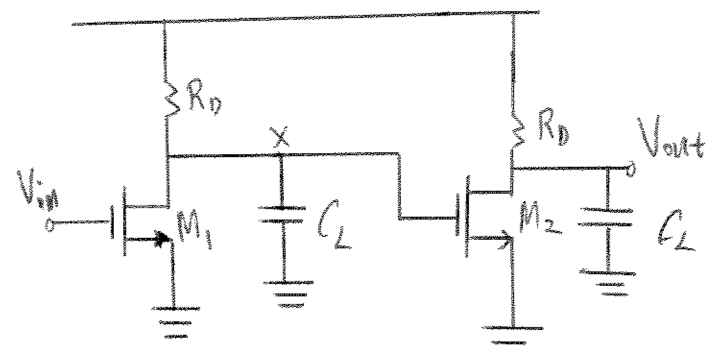
$$\omega_{pin}\,(\omega_{pA}) = \frac{1}{R_G\left(C_{GS1} + C_{GD1}\left(\frac{g_{m1}+g_{m2}+g_{m3}}{g_{m2}+g_{m3}}\right)\right)}$$

$$\omega_{pB} = \frac{g_{m3}+g_{m2}}{\left(C_A + C_{GD1}\left(\frac{g_{m1}+g_{m2}+g_{m3}}{g_{m1}}\right)\right)}$$

$$\omega_{pout} = \frac{1}{R_D\,(C_{DB2} + C_{GD2})}$$

Where $C_A = C_{GS3} + C_{SB3} + C_{GS2} + C_{SB2} + C_{DB1}$

52)



Bias Current = 1mA (each stage)

$C_L = 50 fF$

$\mu_n C_{ox} = 100 \mu A/V^2$, $A_V = 20$, $-3dB$: 1GHz

DC gain: $(g_m R_D)^2 = 20$

$-3dB$ bandwidth: $0.10243/(R_D C_L) = 1 GHz$

Since $C_L = 50fF$, $R_D = 2048.6\,\Omega$

$(g_m R_D)^2 = 20 \Rightarrow g_m = 0.002183 = \frac{2I_D}{V_{eff}} \Rightarrow V_{eff} = 0.916V$

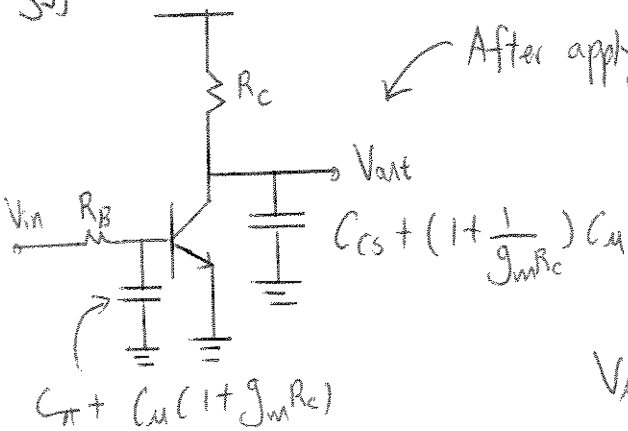
$V_{eff} = V_{GS} - V_{th} = 0.916V$

$g_m = \mu_n C_{ox} \frac{W}{L} (V_{eff}) \Rightarrow \frac{W}{L} = \frac{g_m}{\mu_n C_{ox} (V_{eff})} = 23.83$

So $R_D = 2.05K$, $C_L = 50fF$

$V_{GS} - V_{th} = 0.916V$, $W/L = 23.83$

53)



After apply Miller's theorem

$\omega_{pin} = (2\pi)(500MHz)$

$\omega_{pout} = (2\pi)(2G)$

$I_C = 1mA$, $C_\pi = 20fF$,

$C_\mu = 5fF$, $C_{CS} = 10fF$

$V_A = \infty$.

Low frequency Voltage gain: $\frac{V_{out}}{V_{in}} = \frac{-R_C}{1/g_m + \frac{R_B}{\beta+1}}$

$$\omega_{pin} = \frac{1}{(R_B \parallel r_\pi)(C_\pi + C_\mu(1 + g_m R_C))} = (2\pi)(500\,MHz)$$

$$\omega_{pout} = \frac{1}{R_C[C_{CS} + (1 + 1/(g_m R_C))C_\mu]} = (2\pi)(2G)$$

$$\Rightarrow g_m = 2\pi(2G)[g_m R_C C_{CS} + g_m R_C C_\mu + C_\mu]$$

$$\Rightarrow R_C = \left(\frac{g_m}{(2\pi)(2G)} - C_\mu\right) / (g_m(C_{CS} + C_\mu))$$

$$g_m = \frac{I_C}{V_T} = 0.0386\,\frac{1}{\Omega}, \quad R_C = 5296.53\,\Omega$$

53)

In order to maximize low frequency gain V_{out}/V_{in}, R_B should be as small as possible (restricted by the input pole location). So $R_B \parallel r_\pi \approx R_B$.

$$\omega_{pin} \approx \frac{1}{R_B(C_\pi + C_\mu(1+g_mR_C))} = (2\pi)(500\times10^6)$$

$g_mR_c = 204.446$

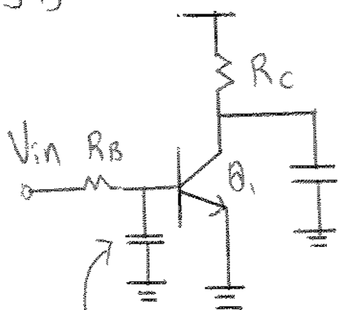
$$R_B = \frac{1}{\omega_{pin}(C_\pi + C_\mu(1+g_mR_c))} \approx 303.95\Omega$$

So

$R_B = 303.95\Omega$

$R_c = 5296.53\Omega$

54)



$C_L + C_{CS} + \left(1+\frac{1}{g_m R_C}\right) C_\mu$

$C_\pi + (1+g_m R_C) C_\mu$

Low freq Voltage gain: $\frac{V_{out}}{V_{in}} = \frac{-R_C}{1/g_m + \frac{R_B}{\beta+1}}$

$$\omega_{pout} = \frac{1}{R_C\left[C_L + C_{CS} + \left(1+\frac{1}{g_m R_C}\right) C_\mu\right]} = (2\pi)(2\,GHz)$$

$$g_m = \frac{I_C}{V_T} = 0.0386\ \frac{1}{\Omega}$$

$$g_m = (2\pi)(2\,GHz)\left[g_m R_C [C_L + C_{CS}] + g_m R_C C_\mu + C_\mu\right]$$

$$R_C = \left[\frac{g_m}{(2\pi)(2G)} - C_\mu\right] / \left(g_m [C_L + C_{CS} + C_\mu]\right)$$

$R_C = 2269.94\,\Omega \approx 2.27\,K\Omega$

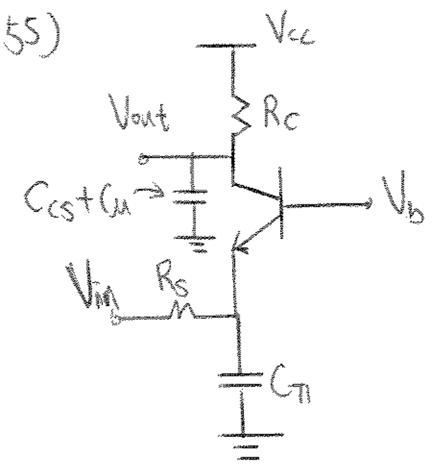
Again, to maximize low freq gain, R_B should be as small as possible, so $R_B // r_\pi \approx R_B$

$$\omega_{pin} \approx \frac{1}{R_B(C_\pi + C_\mu(1+g_m R_C))} = (2\pi)(500\times 10^6),\ g_m R_C = 87.62$$

$R_B = 687.35\,\Omega$

so, $R_C = 2.27\,K\Omega$, $R_B = 687.35\,\Omega$

55)



$V_A = \infty$, $I_C = 1mA$, $R_S = 50\Omega$,
$C_\pi = 20fF$, $C_{CS} = 20fF$, $C_\mu = 5fF$

-3dB bandwidth: 10GHz

Since the output node sees a larger capacitance and resistance than the input, (R_C usually large for large gain), dominant pole and thus -3dB bandwidth occurs at the output.

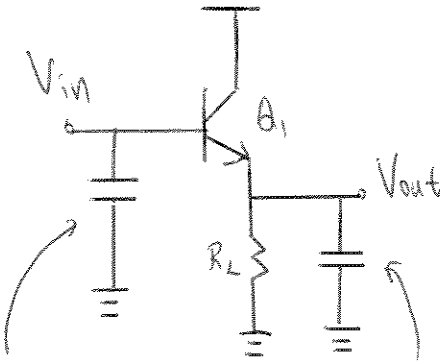
$$\omega_{pout} = \frac{1}{R_C\,[C_\mu + C_{CS}]} = (2\pi)(10GHz)$$

$R_C = 636.62\Omega$, $\frac{1}{g_m} = \frac{25.9mV}{I\,mA}$

Maximum achievable gain: $\frac{R_C}{R_S + \frac{1}{g_m}} = 8.4$

Here we have a tradeoff between gain and bandwidth.

56)



DC gain: $\dfrac{R_L}{R_L + 1/g_m}$

$V_A = \infty$

$C_\mu = 10fF$, $C_\pi = 100fF$

$C_\mu + C_\pi\left(1 - \dfrac{R_L}{R_L + 1/g_m}\right)$

$C_\pi\left(1 - \dfrac{R_L + 1/g_m}{R_L}\right)$

$$C_{in} < 50fF \Rightarrow C_\mu + C_\pi\left(1 - \frac{R_L}{R_L + 1/g_m}\right) < 50fF$$

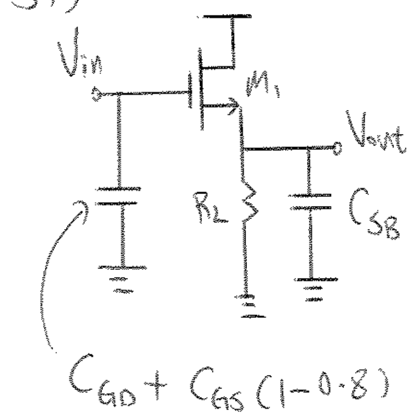
$$10fF + 100fF\left(1 - \frac{R_L}{R_L + 1/g_m}\right) < 50fF$$

$$100fF\left(1 - \frac{R_L}{R_L + 1/g_m}\right) < 40fF$$

$$\left(\frac{\frac{1}{g_m}}{R_L + 1/g_m}\right) < 0.4$$

$$R_L > \frac{3}{2g_m} = 38.85\,\Omega$$

57)



$R_L = 100\Omega$, $I_D = 1mA$

$A_V = \frac{V_{out}}{V_{in}} = 0.8$ $\mu_n C_{ox} = 100\mu A/V^2$

$L = 0.18\mu m$, $\lambda = 0$, $C_{GD} \approx 0$,

$C_{SB} \approx 0$, $C_{GS} = (2/3) WL C_{ox}$

$C_{ox} = 12fF/(\mu m)^2$

$C_{in} = C_{GD} + C_{GS}(0.2)$, $C_{in} = C_{GS}(0.2) = C_{in(min)}$

$A_V = \frac{R_L}{R_L + 1/g_m} = 0.8$, $\frac{1}{g_m} = 25 = \frac{V_{eff}}{2I_D}$

$V_{eff} = 50mV$, $I_D = \frac{1}{2}\frac{W}{L}\mu_n C_{ox}(V_{eff})^2 \Rightarrow W = 1440$

$C_{in,min} = 0.2 C_{GS} = 0.2\left(\frac{2}{3}\right) WL C_{ox} = 414.72fF$

or $C_{in,min} = 0.415pF$

11.58

$$
\begin{aligned}
I_D &= \frac{1}{2}\left(\frac{W}{L}\right)_1 \mu_n C_{ox} V_{ov}^2 = 0.5 \text{ mA} \\
(W/L)_1 &= (W/L)_2 = \boxed{250} \\
W_1 &= W_2 = 45 \ \mu\text{m} \\
g_{m1} &= g_{m2} = \frac{W}{L}\mu_n C_{ox} V_{ov} = 5 \text{ mS} \\
C_{GD1} &= C_{GD2} = C_0 W = 9 \text{ fF} \\
C_{GS1} &= C_{GS2} = \frac{2}{3} W L C_{ox} = 64.8 \text{ fF} \\
\omega_{p,in} &= \frac{1}{R_G\left\{C_{GS1} + C_{GD1}\left(1 + \frac{g_{m1}}{g_{m2}}\right)\right\}} = 2\pi \times 5 \text{ GHz} \\
R_G &= \boxed{384 \ \Omega} \\
\omega_{p,out} &= \frac{1}{R_D C_{GD2}} = 2\pi \times 10 \text{ GHz} \\
R_D &= \boxed{1.768 \text{ k}\Omega} \\
A_v &= -g_{m1} R_D = \boxed{-8.84}
\end{aligned}
$$

59)

$W_2 = 4W_1$, $V_{eff_2} = \frac{V_{eff_1}}{2}$ (To maintain the current constant)

$V_{eff_1} = 200mV$, $V_{eff_2} = 100mV$ (Assume V_{eff_1} is not changed)

DC gain: $-\frac{g_{m_1}}{g_{m_2}} = \frac{-g_{m_1}}{2g_{m_1}} = -\frac{1}{2}$

$$\omega_{pin} = \frac{1}{R_G\left[\frac{2}{3}WLC_{ox} + (0.2)W\left(\frac{1}{2}\right)\right]} = (5\times10^9)(2\pi)$$

$W = 45\mu$

$\Rightarrow R_G = 459.32\,\Omega$

$$R_o = \frac{1}{(10\times10^9)(2\pi)(0.2)(4)(45)} = 442.097\,\Omega$$

DC gain: $|g_{m_1}R_o| = \frac{2I_D}{V_{eff_1}}R_o = 2.2105$